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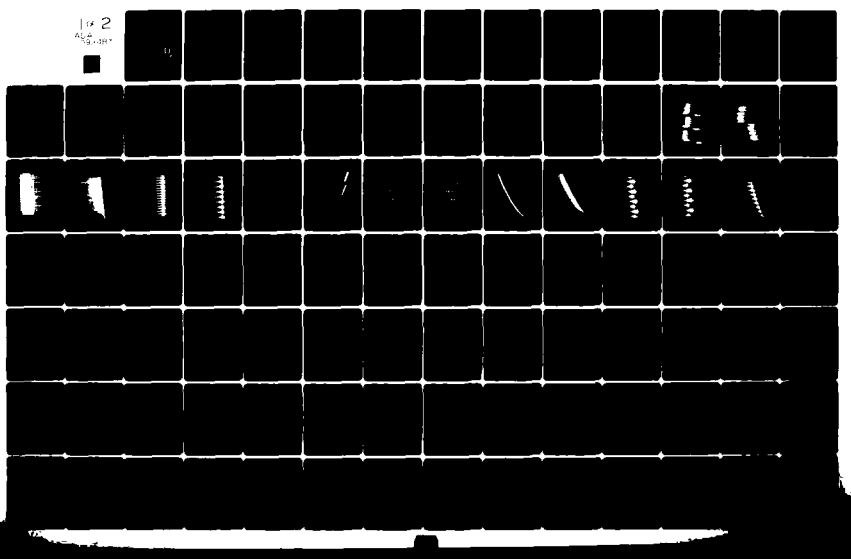

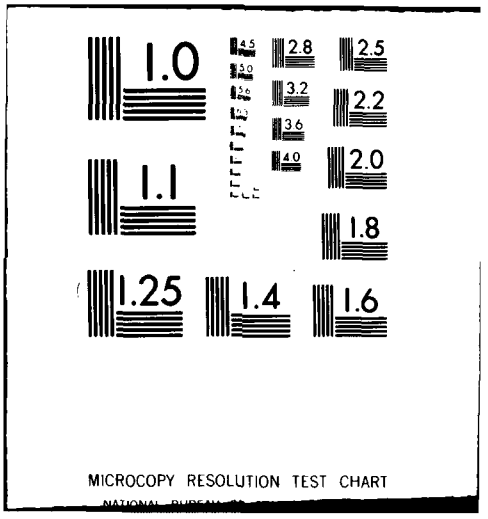
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November 1980

Fluidic Heading System Study: Summary Report

By Abraham Finkel

Prepared by

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The feasibility of incorporating a fluidic angular rate sensor into a vehicle heading sensor system is investigated. A heading sensor of this type would find application in armored vehicles where a magnetic compass cannot be used. The study covers system considerations, microprocessor-based computational requirements, and the effects of rate sensor errors.

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1. INTRODUCTION

1.1 Statement of Task

The purpose of this study was to investigate the feasibility of incorporating a fluidic angular rate sensor into a heading sensor system. The fluidic sensor may offer a cheap and rugged alternative to the rate gyro. A heading sensor of this type would find application in armored vehicles where a magnetic compass cannot be used. Goals for the system are low production cost (< $6,000) and a drift rate of 0.8 deg per hour, although a higher drift rate may be acceptable in some applications.

1.2 Principal Conclusions and Recommendations

(a) The fluidic sensor (including necessary fluidic amplifiers) demonstrates sensitivity of the required order of magnitude.

(b) Drift characteristics under laboratory conditions are encouraging.

(c) A key element (a transducer which converts pressure to an electrical signal with the necessary stringent requirements) is yet to be demonstrated and developed.

(d) Many important environment characteristics are yet to be determined with precision.

(e) Essential environmental tests should be applied to an integrated sensor-transducer package.

(f) Certain effects such as the long-term equivalent drift under vibration are best measured by integration of the output. A high-quality continuously rebalanced integrator should be developed. An integrator of this type will likely be required to furnish angular incremental data to the computer.

(g) Modern microprocessors can handle the computation burden. Thirty-two bit precision will be required with a sampling frequency of about 100 per second.

(h) The main thrust of further effort should be directed toward the construction of an integrated sensor-transducer package before conducting precision environmental tests.

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2. SYSTEM CONSIDERATIONS

2.1 Description

For a vehicle whose orientation is perfectly horizontal (no pitch or roll), a heading may be obtained by simply integrating the angular rate of the vehicle's vertical axis. Hence, for such a single axis system, a single angular rate sensor is sufficient. With pitch and roll present, large errors would result and it is necessary to measure angular rates about all three vehicle axes. The required computations are, of course, much more complex and are briefly outlined in Appendix A.

At some point, digitizing is required so that computation may proceed with the necessary accuracy. It would be possible to sample the angular rate sensors directly and enter rates into the computer. Noise and vibration are apt to make this approach unworkable unless strong filtering is employed, in which case vital rate information may be lost. Furthermore, a rate resolution equal to the desired drift rate of 0.8 deg/hr (0.0002 deg/s) is required. If the vehicle's maximum angular rate is 40 deg/s, an 18-bit analog-to-digital conversion is implied for each sample.

A more practical approach utilizes angle increments as the basic unit. Angle increments are obtained by integrating the rate sensor output and sampling the integrator output. Rapid sampling is still required to keep the increments small, since knowledge of varying velocity between samples will be lost. The computational algorithm assumes constant velocity between samples, although it is possible, with added complexity, to estimate acceleration and higher-order terms utilizing more than one sample interval. The integration which takes place prior to digital computation effectively filters noise and properly reacts to high-frequency motion, thus giving correct angle increments. A carefully constructed pulse-balanced integrator can circumvent the need for 18-bit readout while maintaining 18-bit sensitivity to any accumulated angular increments by virtue of the memory aspect of the integrator capacitor. The features of this type of circuit will be described at the end of this section.

A system would therefore take the form illustrated in Figure 1.

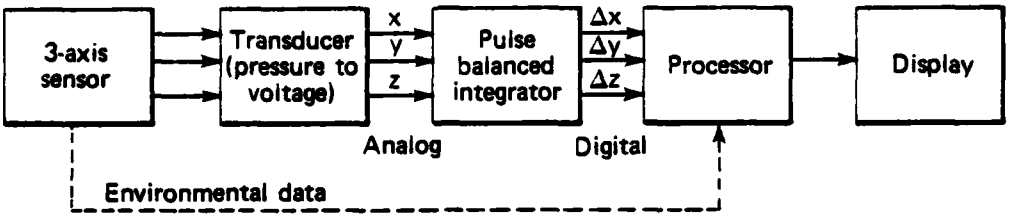

Figure 1 Heading sensor system block diagram.

The transducer need not be limited to a voltage output as shown. The integrator block would be modified to accommodate other types. In particular, if a pressure-to-frequency transducer were available, the integrator would simply be a counter. To the extent that the sensor-transducer combination cannot be sufficiently isolated from the environment, an additional data path to the computer is shown to allow for proper compensation. (Of course, transfer functions must be stable and known with precision.)

2.2 Operation

The operator will put the computer into one of two modes: ALIGN or NAVIGATE. In the align mode, start-up information must be fed into the computer so that it can calculate initial values for the elements of the attitude matrix and earth-rate components. Entry could be made via either a simple keyboard or a number of switches. The required input data are latitude and pitch, roll, and heading angles.

Latitude must be known so that the effect of earth rate may be properly accounted for. Pitch and roll angles may be read from gravity-sensitive devices (e.g., a gimballed platform with level indicators). Initial heading must be determined with reference to a known direction. Note that the computer will not properly calculate departure from an arbitrarily defined initial heading. The absolute heading must be known to enable calculation of earth rate effects. The gyrocompassing technique of determining initial alignment, conventionally used in inertial navigation systems, requires very accurate sensing of earth rate and does not appear to be within reach for the present fluidic sensors.

With the initial alignment procedure complete, the stationary vehicle should sense no motion with respect to the ground. Nonzero outputs are due to sensor bias, and an observation period is required to determine the magnitudes (and rate of change, if uniform). Bias (and bias rate) are thus known corrections to be applied during the NAVIGATE mode. Advantage could be taken of any subsequent vehicle stops to obtain updates on the sensor bias.

The computer is switched to NAVIGATE prior to the start of vehicle travel. Updating of the attitude matrix and display of heading then proceeds automatically.

2.3 Microprocessor

The computational requirements for one particular algorithm are outlined in Appendix A. Thirty-two bit precision is required, with updating at a rate of about 100/s. A 16-bit microprocessor is a logical choice with the possible use of a hardware multiplier if execution time must be improved. Variations from a basic configuration to provide various compensations will not have major impact on either cost or execution time (e.g., look-up tables to modify the transfer function). A significant simplification could be made if no compensation at all were required, for then power-of-two angular increments could be used as multipliers, resulting in simple shift operations. Some simplification of the algorithm may also be possible by changing the reference coordinate system, as described in Appendix B.

For high-volume production, development and programming costs need not be considered. The cost of a fully packaged microprocessor with the program contained in masked read-only memory (ROM) will probably lie in the $1- to $2-K range. As an example, a prewired and assembled Texas Instruments module (TM 990/101M) containing memory is available for $500 to $600 in production quantities. The microprocessor component alone (TMS 9900) is available for $30. A full-temperature military version (SBP 9900AM) costs $400.

2.4 Integrator

Considerable dynamic range must be exhibited at the output of the angular rate sensor. The sensor must be capable of responding to both the maximum angular rate ($\approx$ 40 deg/s) and the allowable drift rate ($\approx$ 0.0002 deg/s), a range equivalent to 18 bits. However, if integration precedes the computer, sampling of the integrator output requires only that the readout resolution be sufficient for accurate updating. Thus, if 10 ms

sampling is used, the maximum angular step is 0.4 deg, while a resolution of 0.04 deg may be sufficient provided that the remaining angle increment less than 0.04 deg is retained and added to the next increment. Despite the coarse angle readout, there is no accumulation of errors. These properties are exhibited by the pulse-rebalanced integrator shown in simple form in Figure 2.

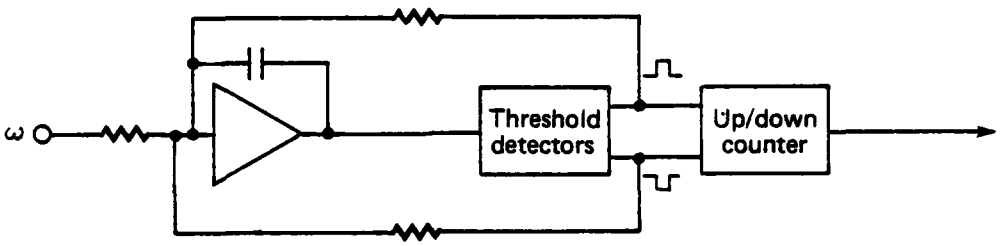

Figure 2 Pulse-rebalanced integrator.

The integrator output represents angle. When a positive or negative threshold equal to the least significant readout bit is reached, a precision pulse is generated to rebalance the integrator. The change of integrator output voltage due to the pulse must equal this resolution angle. The pulses drive a counter which is read out periodically by the processor.

While the readout need only consist of a few relatively coarse bits, the bias and drift at the integrator input must still satisfy the 18 bit requirement. This is readily achievable with good quality modern operational amplifiers and careful design. Changes of components or charge quantities result in scale factor errors. Scale factor stability of 0.1 to 1% is tolerable provided that such changes apply uniformly to all segments of the transfer function. Innovative circuit design will be required to guarantee equal scale factors for positive and negative inputs ($\approx 0.01\%$).

This circuit is in reality a voltage-to-frequency converter with the resulting pulses counted to develop angle increments and will be necessary to accommodate a pressure-to-voltage transducer. If a pressure-to-frequency transducer of sufficient accuracy is developed, then, of course, only a counter would be required.

2.5 System Limitations

The heading sensor system cannot calculate position unless linear velocity (or acceleration) is also measured. Hence, the change of attitude due to travel over the earth's curved surface must be neglected.

Travel over level ground of 70 miles from the starting point results in an apparent tilt of 1 deg. Bias errors also develop, since the effect of earth rate is not properly compensated. In particular, a north-south travel of 70 miles from the equator will cause the vertical axis to sense an uncompensated rotation rate of 0.25 deg/hr.

For missions of up to three hours, therefore, these errors will have little impact on system accuracy.

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3. COMPUTER STUDIES OF SENSOR REQUIREMENTS

3.1 Purpose

It is relatively easy to see the effects of sensor errors in a single-axis system. A scenario of angular motion is multiplied by the normalized transfer function of the sensor (output vs. angular rate). The product is integrated over time to obtain the angle output. This may be compared to the precise angular position which is given by the integral of the input angular motion. Hence, for example, a perfectly linear, stable transfer function with an unanticipated offset of 1 deg/hr will simply accumulate an angular error at the rate of 1 deg/hr, regardless of the input scenario. Nonlinear transfer functions produce additional errors which are scenario-dependent due to a rectification effect which is equivalent to adding additional bias or offset.

For the full 3-axis system, there will be additional errors due to cross-coupling. It was always felt that, given a system which is not allowed to produce errors greater than, say 3 deg (1 deg/hr for a 3 hr mission), the 3-axis effects would not be drastic and that the simple one-axis system could be the basis of rule-of-thumb requirements on the sensors. However, to demonstrate the correctness of the intuitive conclusions, a computer simulation was performed. This simulation allowed for sensor bias and scale factor errors only. "Bias" should be considered to be due to all contributory factors: unpredictable offset, the effect of motion scenario upon an asymmetrical or nonlinear transfer function, and the rectification effects of vibration (discussed further in Section 6).

The following requirements for a successful computer analysis are somewhat incompatible:

3.1.1 Long simulated (vehicle) time

If 3-hr missions are contemplated, the full 3 hr should be simulated. Heading error may grow nonlinearly, and it would be incorrect to extrapolate the results of a shorter run.

3.1.2 Reasonably short computer time

If a multitude of runs is to be made so that many factors may be varied, a short run time is essential.

3.1.3 Highly accurate attitude updating

Typical similar computer studies derive "system" errors rather than errors due solely to sensor deficiencies. Contributing to these system errors are also the approximation errors inherent in the attitude-updating procedure. Accuracy is obtained by high-frequency updating which, of course, requires long computer runs and, even then, roundoff errors may become a factor.

3.2 Approach

The foregoing incompatible requirements may be satisfied through a slight subterfuge. The underlying vehicle scenario is described in terms of sine waves (a convenient mathematical representation is needed to enter 3-hr worth of motion into the computer). These sinusoidal waveforms are then quantized at sampling intervals which are short enough to preserve the basic nature of the motion; i.e., many samples per cycle. It is this quantized motion, a series of angular steps, which then becomes the assumed angular motion applied to the vehicle. Computer updating need be performed only when the angular steps occur. The three principal sources of computation errors are either eliminated or minimized:

(1) There is no varying velocity between updates since there is no vehicle motion between steps.

(2) The solution of the matrix differential equation required by the updating procedure involves a matrix exponential. In the normal algorithm, one or more terms of the series approximation will be used. With a large-scale computer and the relatively infrequent updating required by the quantized motion, an exact closed-form solution may be used.

(3) Computer roundoff error is considerably reduced due to the reduced frequency of updating.

To further reduce the total number of updates, only angular steps exceeding a prescribed minimum are allowed. If, at a sampling instant, the angular step were less than the minimum, no step at all would be allowed. Thus, if the scenario includes long periods of little or no motion, fewer compulations are made, thereby decreasing both the errors and computer time.

As described elsewhere, a nine-element direction cosine (or "C") matrix describes the vehicle attitude. In this computer program, two such C matrices are updated at each angular step. The reference matrix, CREF, describes the actual vehicle attitude. The computed matrix, CCMP, describes the attitude obtained based on erroneous data from the rate sensors. Since heading error is of primary interest, heading is calculated from each of the matrices with the difference tabulated as the heading error. Whenever an angular step is encountered, the following three C-matrix updates are performed:

(1) CCMP is updated for any rotation sensed since the previous angular step. A sensor bias will, of course, result in a fictitious rotation. Also, since the matrix is misaligned, earth rotation is not properly compensated for so that the computer thinks there has been some net motion with respect to the ground.

(2) CREF is updated for the angular step, involving one, two, or all three axes.

(3) CCMP is similarly updated for the angular step. In this case, the angular steps reflect the erroneous scale factors assumed for the angular rate sensors.

The exact, closed-form solution of the matrix differential equation is shown in Appendix C, along with an equivalent formulation convenient for computation.

3.3 Program Inputs and Outputs

To allow flexibility of vehicle scenario, angular motion of each of the three axes is specified in terms of angular amplitude and period for each of four successive intervals. The time at the end of each interval may also be selected. Any number of iterations of the four time intervals may be chosen to simulate any total vehicle time. Additionally, in one version, a steering instability (or Z-axis fluctuation) was superimposed.

The program as written also allows selection of latitude, initial heading, sampling interval, minimum angular step, and printout interval.

Output quantities tabulated are time, heading, tilt, and heading error. Since pitch, roll, and yaw are independently specified, the resulting attitude is fully determined. As

further discussed in Appendix D, symmetrical motion about the vehicle axes can result in a considerable attitude "creep." The tilt readout shows whether this effect is getting out of hand, resulting in an unrealistic scenario although, in any event, the ability of the computer to follow with acceptable error is the concern of these studies.

The final two columns of the readout are labelled DETCREF and DETCCMP. These list at each time the value of the determinants of the respective matrices. For many of the C-matrix updating procedures, a prime concern is the development of "skewness" or lack of orthogonality as updating proceeds. A property of an orthogonal matrix is that the value of its determinant is unity. The printout of DETCREF and DETCCMP is one way of checking that the matrix is being updated properly. A listing for this program is given in Appendix E. A selected number of computer printouts is reproduced in Appendix F.

3.4 Tabulation of Results

A quick look at the results under various conditions may be obtained by examining the heading at the end of 3 hr. For a fair comparison, we wish to exclude that portion of the error which is simply the final heading excursion multiplied by the scale factor error. Thus, if at a heading of 100 deg a system with a 2% scale factor error shows a heading error of 2 deg (of comparable sign), then there has been no error accumulation. The latitude is 45 deg unless otherwise indicated.

3.4.1 Z-axis motion only: continuous half sine waves of 90 deg amplitude repeating every 10 minutes

(a) Z sensor scale factor 1.02; 1 deg/hr bias

accumulated heading error after 3 hr = 2.08 deg

(b) Z sensor bias error only of 1 deg/hr

heading error = 2.85 deg

(c) Z sensor bias error only of 1 deg/hr; latitude = 0 deg

heading error = 2.70 deg

(d) Z sensor bias error of 1 deg/hr; latitude = 90 deg

heading error = 3.00 deg

[shows that at 90 deg LAT, where the Z axis is aligned to the earth's spin axis, a misalignment due to Z-axis rotation does not pick up additional errors due to earth rate. Also, the computation is exact.]

(e) Z sensor bias error of 1 deg/hr; sampling interval reduced from 1.0 to 0.5 s

heading error = 2.85 deg (same as (b))

(f) All sensors with bias of 1 deg/hr and scale factors of 1.02

heading error = 1.06 deg

(change of ≈ 2 deg due to cross axis effects)

(g) Reduced Z amplitude of 5 deg; 0.1 deg angle no sensor errors

heading error = 0

(h) Reduced Z amplitude of 5 deg; 0.1 deg steps; Z sensor bias of 1 deg/hr

heading error = 2.85 deg (same as (b))

(i) Reduced Z amplitude of 5 deg; 0.1 deg steps; all sensors with 1 deg/hr bias and scale factors of 1.02

heading error = 1.30 deg

(j) Full Z amplitude; X, Y, Z scale factors 1.02; X, Y, Z biases of -1, -1, +1 deg/hr

heading error = 2.91 deg

(compare to (f); this choice of signs has increased error)

3.4.2 Full Scenario: Three Axes Motion

1st 100 s - X=10 deg amplitude, 10 s period
Y=10 deg amplitude, 11 s period
Z=90 deg amplitude, 1200 s period

next 100 s - no Y motion
next 100 s - no X motion
next 300 s - no X nor Y motion

(pattern repeats every 600 s)

1.0 s sampling
1.0 deg minimum angle step

(a) Z sensor bias of 1 deg/hr

heading error = 2.83 deg

(almost identical to 3.4.1(b); no cross-axis effect with perfect X,Y sensors)

(b) All sensors with 1 deg/hr bias

heading error = 1.84 deg

(c) Bias errors (X, Y, Z) of -1, -1, +1 deg/hr

heading error = 3.71 deg

(d) Scale factors of 1.02 on all sensors; no bias

heading error = -1.07 deg

(e) Scale factors of 1.05 on all sensors; no bias

heading error = -2.73 deg

(f) X, Y, Z scale factors of 0.95, 0.95, 1.05, no bias

heading error = -2.60 deg

(g) Combined sensor errors: scale factor 1.02 and bias of 1 deg/hr on all sensors

heading error = 0.78 deg

(Note: (b) + (d) = 0.77 deg)

(h) Combined sensor errors: 1.02 scale factor on all sensors; X, Y, Z bias of -1, -1, +1 deg/hr

heading error = 2.65 deg

(i) Repeat of (g) with minimum step of 0.5 deg

heading error = 0.97 deg

(j) Repeat of (g) with minimum step of 0.5 deg, 0.5 s sampling

heading error = 0.97 deg

3.4.3 Full scenario changed to eliminate simultaneous pitch and roll (X motion removed during 1st 100 s). Therefore, no Z axis coning motion. Sensor errors as in 3.4.2(g)

heading error = 0.47 deg

3.4.4 Full scenario changed to reflect higher frequencies as shown on charts of measured pitch and roll

A chart for one particular vehicle course shows high pitch and roll amplitudes of 10 deg and 5 deg, respectively. Those motions are in phase with a period of 2.5 s. A worst-case is simulated where this motion continues for the entire 3 hr. A single half-sine wave of yaw having 90 deg amplitude and a 200-s half-period is assumed every 600 s. In addition, a yaw fluctuation or steering wander is assumed to be continuously present with 3 deg amplitude and a 3 s period. The sampling interval is reduced to 0.3 s to accommodate these higher frequencies.

(a) Z sensor bias of 1 deg/hr

heading error = 2.90 deg

(compare to 3.4.2(a) = 2.83 deg)

(b) Z sensor scale factor 1.02

heading error = -1.48 deg

(c) X, Y, Z bias errors of -1, -1, +1 deg/hr

heading error = 3.63 deg

(compare to 3.4.2(c) = 3.71 deg)

(d) Combined sensors errors: scale factor 1.02 and bias of 1 deg/hr on all sensors

heading error = 2.35 deg

(compare to 3.4.2(g) = 0.78 deg)

3.4.5 Full scenario changed to include coning motion

Since in-phase motion does not produce the serious coning effects of quadrature motion, the roll (Y) period was changed to 2.6 s, thereby giving a continuously varying phase difference between X and Y. There is now considerably greater creep of the vehicle both in heading and tilt. (See discussion of this effect in Appendix D). While somewhat unrealistic, the heading sensor system should still be able to follow with reasonably small error.

(a) Z sensor bias of 1 deg/hr

heading error = 2.89 deg

(b) Scale factor 1.02, bias of 1 deg/hr on all sensors

heading error = 4.58 deg

(c) Repeat of (b) but with steering wander removed

heading error = 4.70 deg

3.4.6 Full scenario changed to test for highest frequency pitch and roll

Most of the measured vehicle motion shows pitch and roll motion of 3 deg amplitude, 1.5 s period. An additional program was run with continuous X axis and Y axis motion of 3 deg amplitude and periods of 1.5 and 1.6 s, respectively. Yaw motion remained as in 3.4.4. The sampling interval was further reduced to 0.2 s. With scale factors of 1.02 and biases of 1 deg/hr on all axes:

heading error = 1.80 deg

3.5 Conclusions

The data substantiated the tentative conclusion stated earlier; namely, that for systems with allowable errors of only a few degrees, the contributions of cross-axis effects will be minor. In none of the cases where scale factor errors were limited to 2% did the heading error after 3 hr exceed the simple single-axis calculation by more than 2 deg (single-axis heading error = Z-axis bias times 3 hr). A case of 5% scale factor errors (2e) did produce a heading error of 2.73 deg, but a 5% scale factor error may already be considered intolerable, since the heading will always be in error by 5% X angular excursion. Thus, anytime the vehicle has turned 180 deg, an error of 9 deg will be encountered.

The foregoing analyses included only two of many types of sensor errors: bias and scale factor. Some other types of sensor errors such as asymmetry and nonlinearity may be examined in terms of bias and scale factor (see Section 6). The effects of g-sensitivity could be entered into this program if a realistic profile of vehicle acceleration could be modeled.

4. COMPUTER STUDIES OF MICROPROCESSOR BASED ALGORITHMS

4.1 Purpose

The results of Section 3 indicate heading errors due to various combinations of sensor errors and scenarios, processed by near-perfect computations. This section addresses the corollary question: Assuming perfect sensors, will a practical microprocessor achieve the desired accuracy? The algorithm utilized requires periodic updating of the nine-element direction cosine matrix as described in Appendix A. Alternate computational techniques were not considered since feasibility rather than optimization was the objective.

4.2 Approach

4.2.1 Reference Program

As in Section 3, an appropriate scenario must be selected since errors will be scenario-dependent. Following somewhat the procedures used in that section, a basic 600 s time span was divided into intervals of 192 and 408 s. A yaw motion of a full half sine wave (amplitude = 1.5 radian and period = 384 s) is imposed during the first 192 s, followed by no yaw during the balance of the 600-s span. Throughout the 600-s span, sinusoidal pitch and roll motions of 0.06 radian amplitude are assumed. The pitch and roll periods are 1.5 and 1.6 s, respectively. As noted in Section 3, a choice of slightly different pitch and roll frequencies allows varying phase relationships.

A reference program, referred to in the program listing as the tank simulation program (TNKSIM), accepts the foregoing scenario and generates heading, tilt, and a record of angular increments to be sensed by the physical sensors, with earth rate accounted for. These angular increments are subsequently used as the inputs to the "test" program. Due to the lengthy time required, the program allows for follow-on runs to previous records by reading the last attitude matrix as a starting point. A succession of such runs provided a record of 30-min of vehicle travel.

Since maximum accuracy is an objective of the reference program, calculations proceed with full precision and with 2 ms updating. A second-order algorithm (see Appendix A) was utilized with parallel updating (wherein all values of the

present instant are obtained solely from values of the previous instant). Skewness of the attitude matrix was monitored by printing the value of its determinant. After 30 min, a departure from unity of 2×10^{-8} was obtained. A further test of the reference algorithm accuracy is described in Appendix G.

4.2.2 Test Program

In this program (TNKTST), the reference record is the source of sensor inputs. The program models the pulse-balanced integrator to develop periodic angular increments which are then used as the source of updating. A second-order algorithm identical to that used for the reference program is used, except that now computational limitations are intentionally inserted. For the two sets of runs completed, precision of 32 and 24 bits were selected. In both cases, a 10-ms update interval was used, coupled with an integrator threshold (least significant bit) of 0.06 deg. Heading and tilt are calculated and compared to the reference to obtain errors.

4.2.3 Plots of Outputs

Plots of heading, tilt, heading error, and tilt error were obtained by further sampling of the computed outputs. To plot 30 min of data requires a sampling frequency well below the frequency of the fine structure. Therefore, care must be exercised in interpreting fine structure of the plots (as well as the more frequently sampled data available in printouts). The low-frequency components are of much more significance.

Figures 3 through 11 were obtained with 32 bit computation. Heading (as obtained by the 32 bit processor) and heading error for the entire 30 min are shown in 3 and 4. Expanded plots of part of the error record are shown in 5 and 6. The effect of the 0.06 deg integrator threshold is clearly seen. Calculated tilt angle for the entire period is shown in 7. Tilt error for the entire period is shown in 8, with expanded plots in 9 and 10.

Figures 11 through 19 are similar to the previous plots, except for 24-bit computation. Figure 16 (as well as Figure 8) seems to show a discontinuity at 200 s. The expanded plot of 19 demonstrates that this is merely a quirk of insufficient resolution coupled with the piecewise construction of the scenario. The apparently missing 2-deg fluctuations are now made visible.

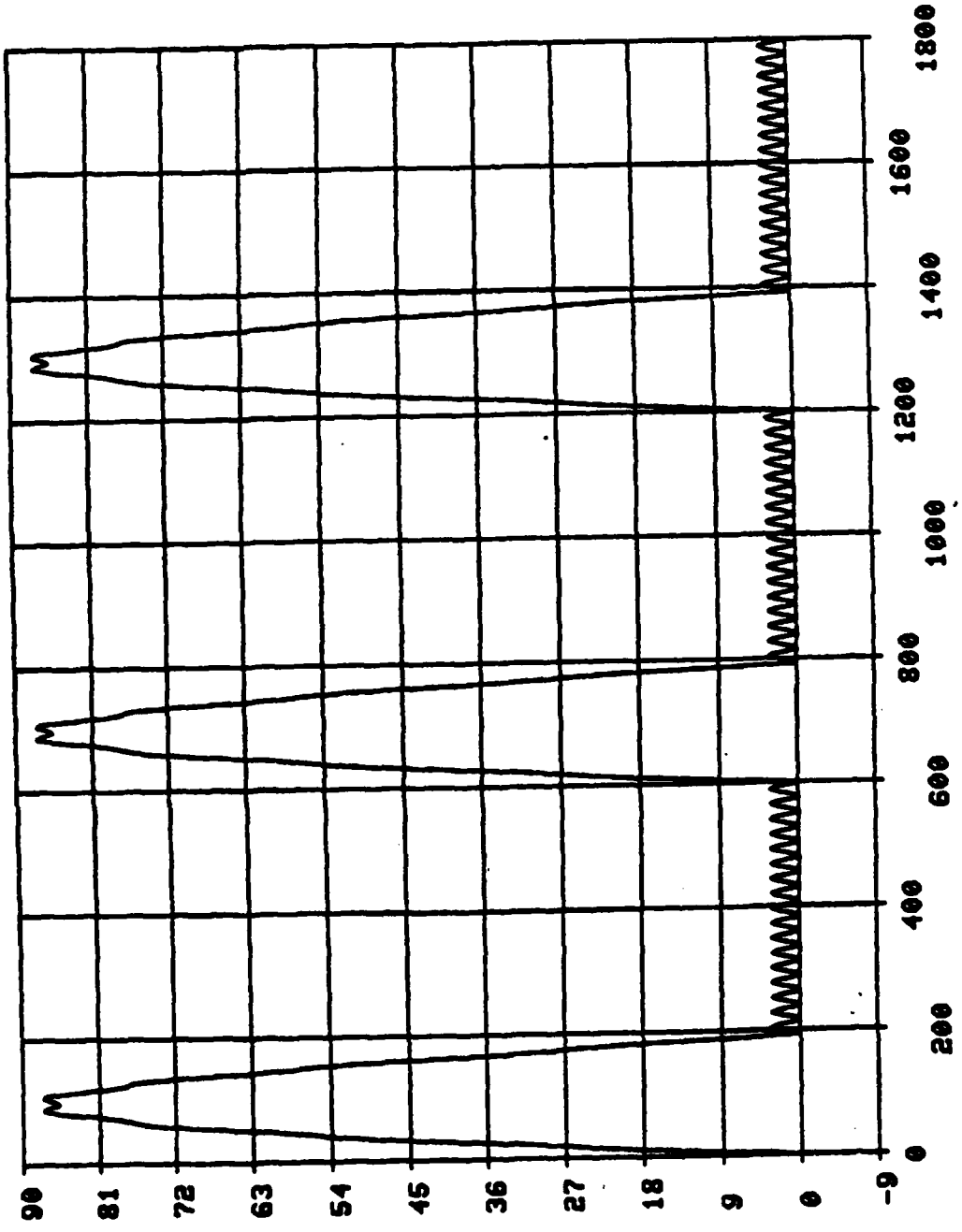

Figure 3. Heading obtained with 32 bit computation.

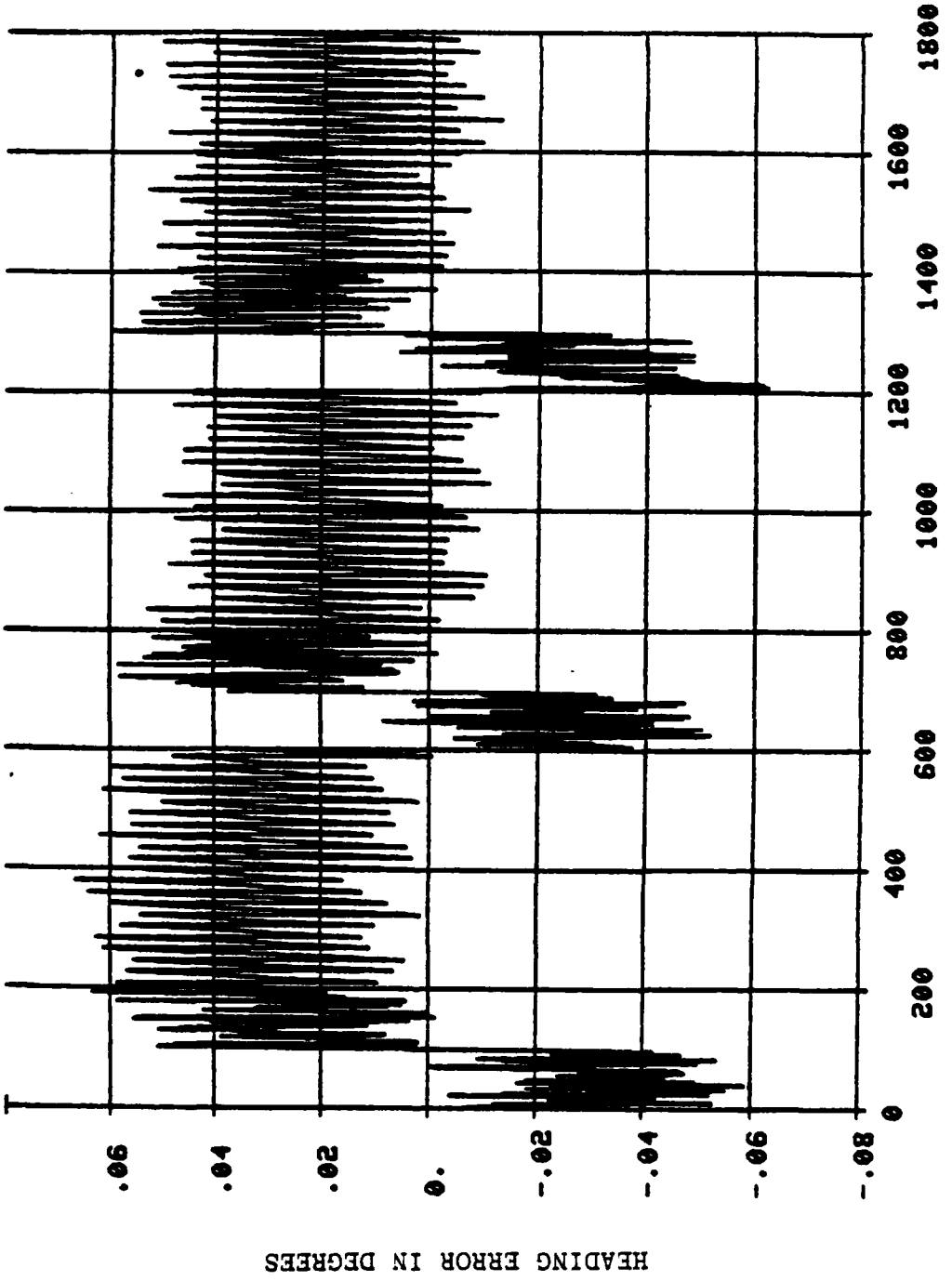

Figure 4. Heading error obtained with 32 bit computation.

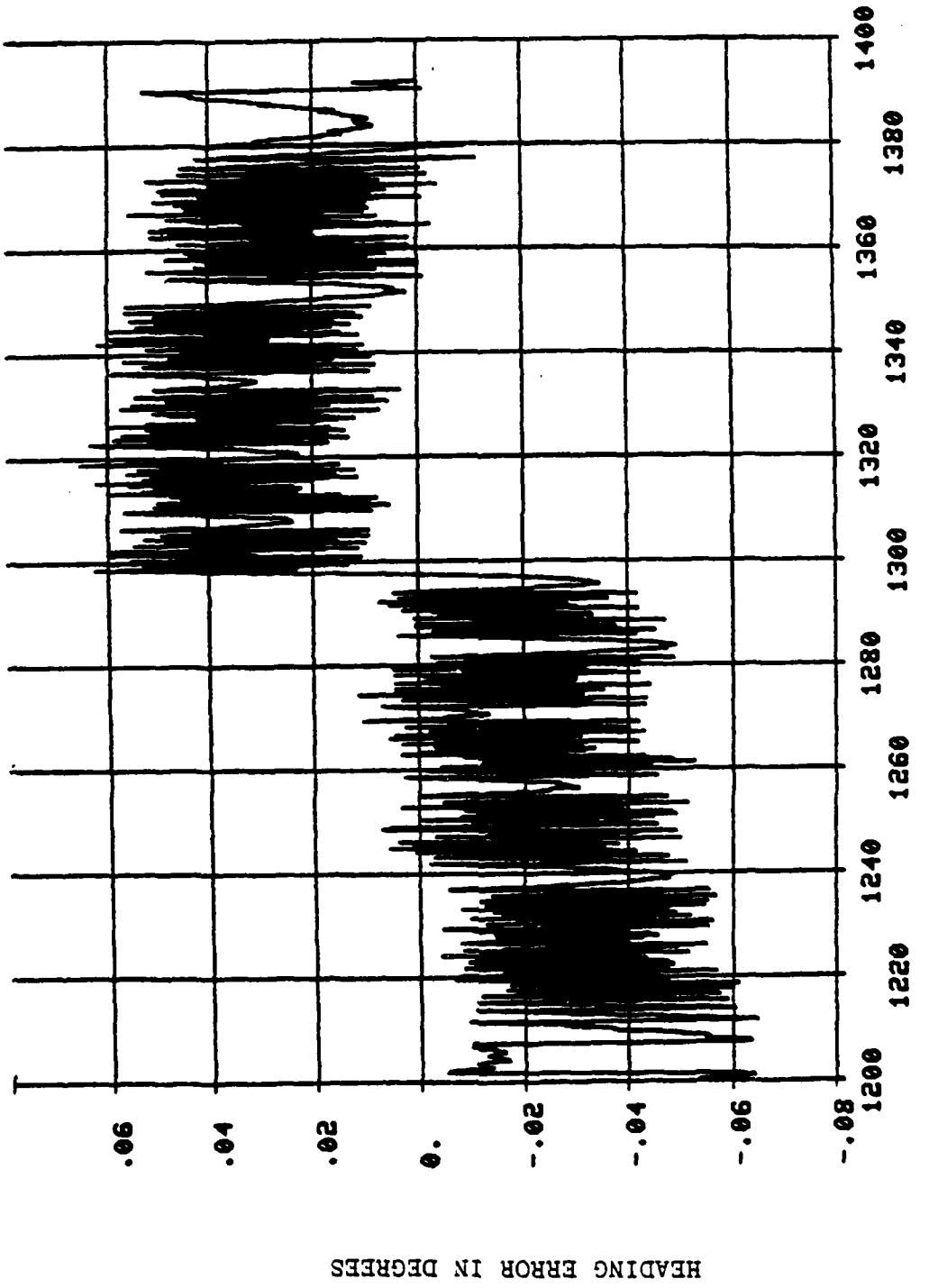

Figure 5. Heading error obtained with 32 bit computation (expanded time scale).

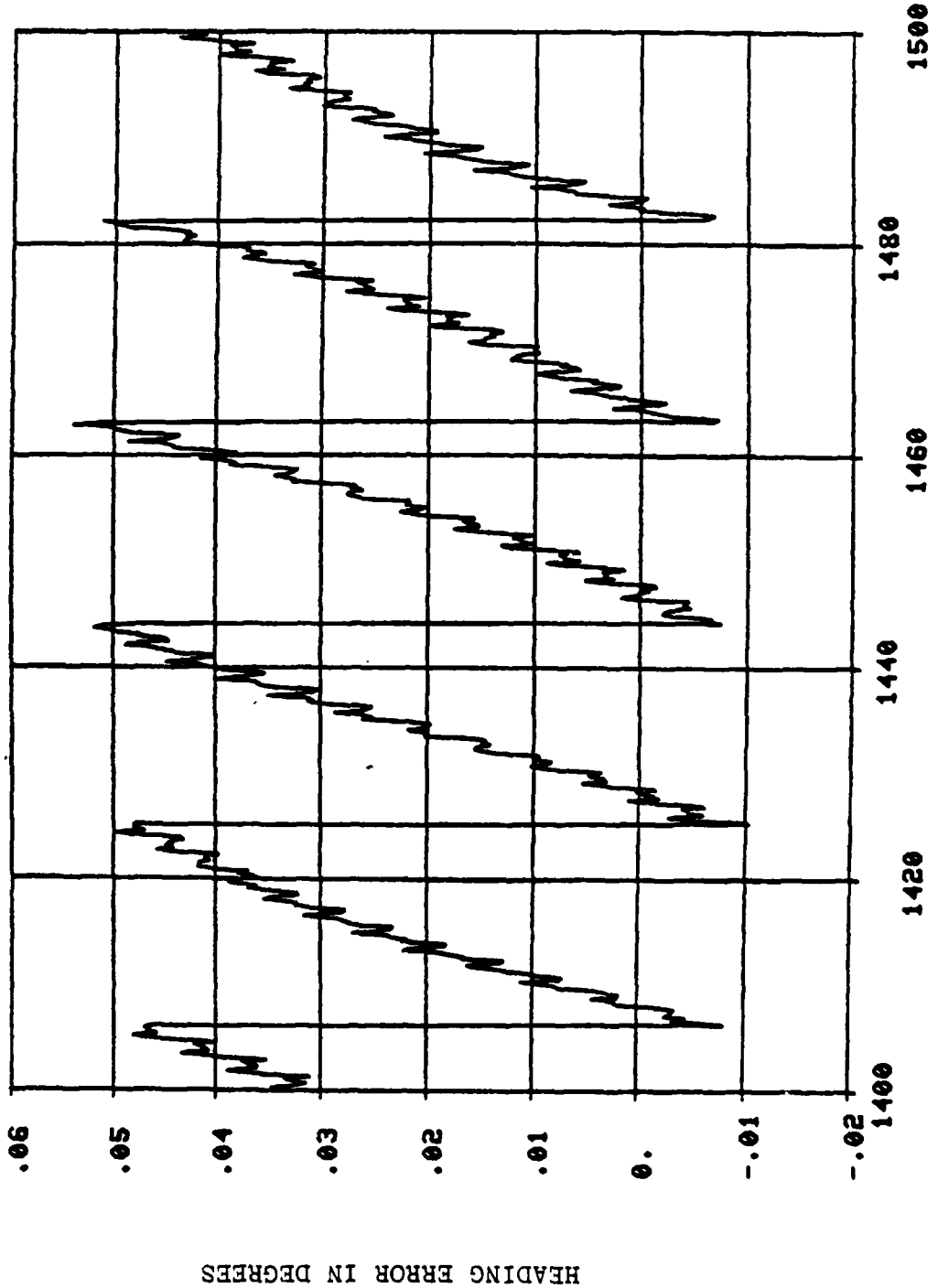

Figure 6. Heading error obtained with 32 bit computation (expanded time scale).

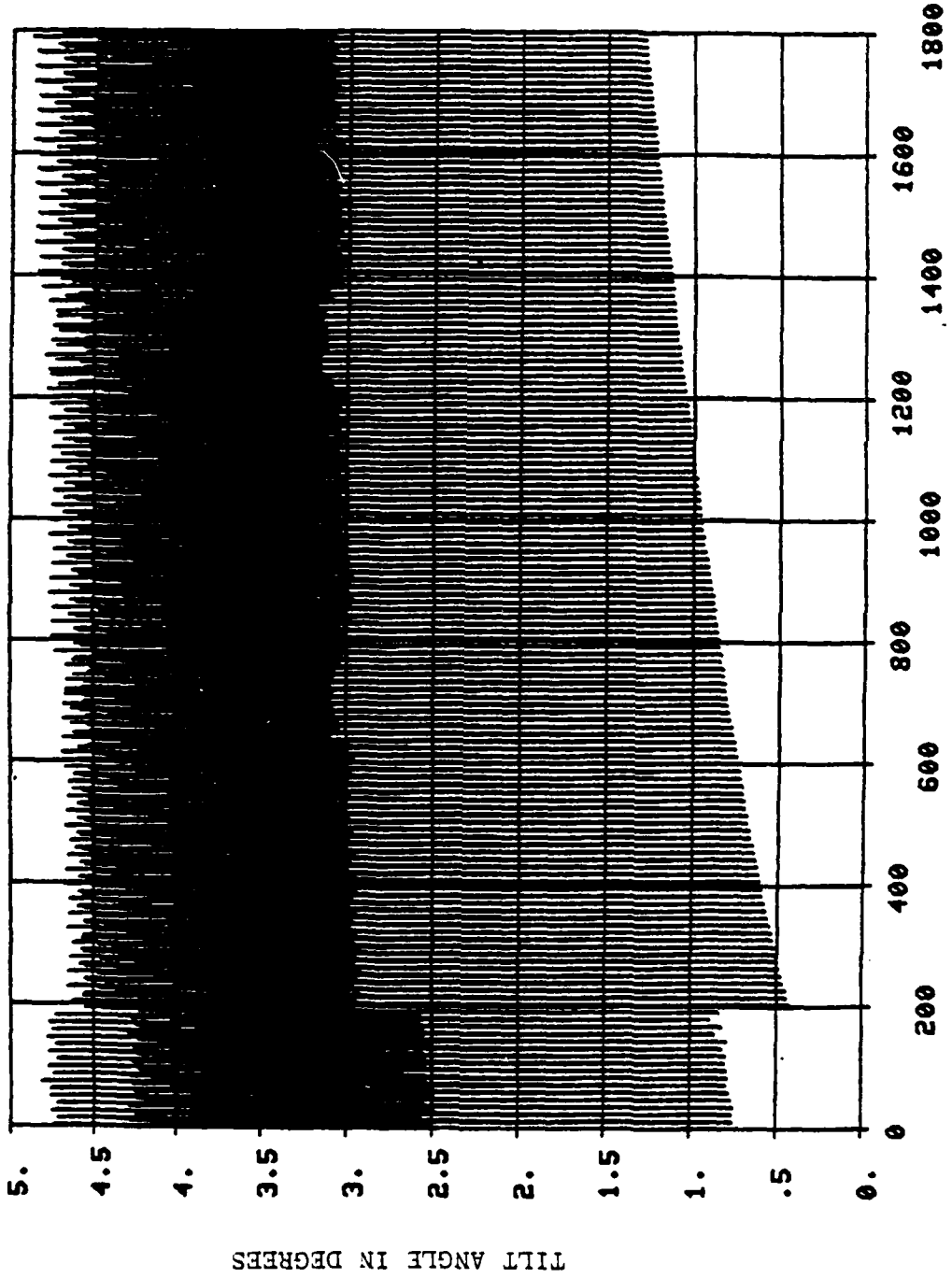

Figure 7. Tilt angle obtained with 32 bit computation.

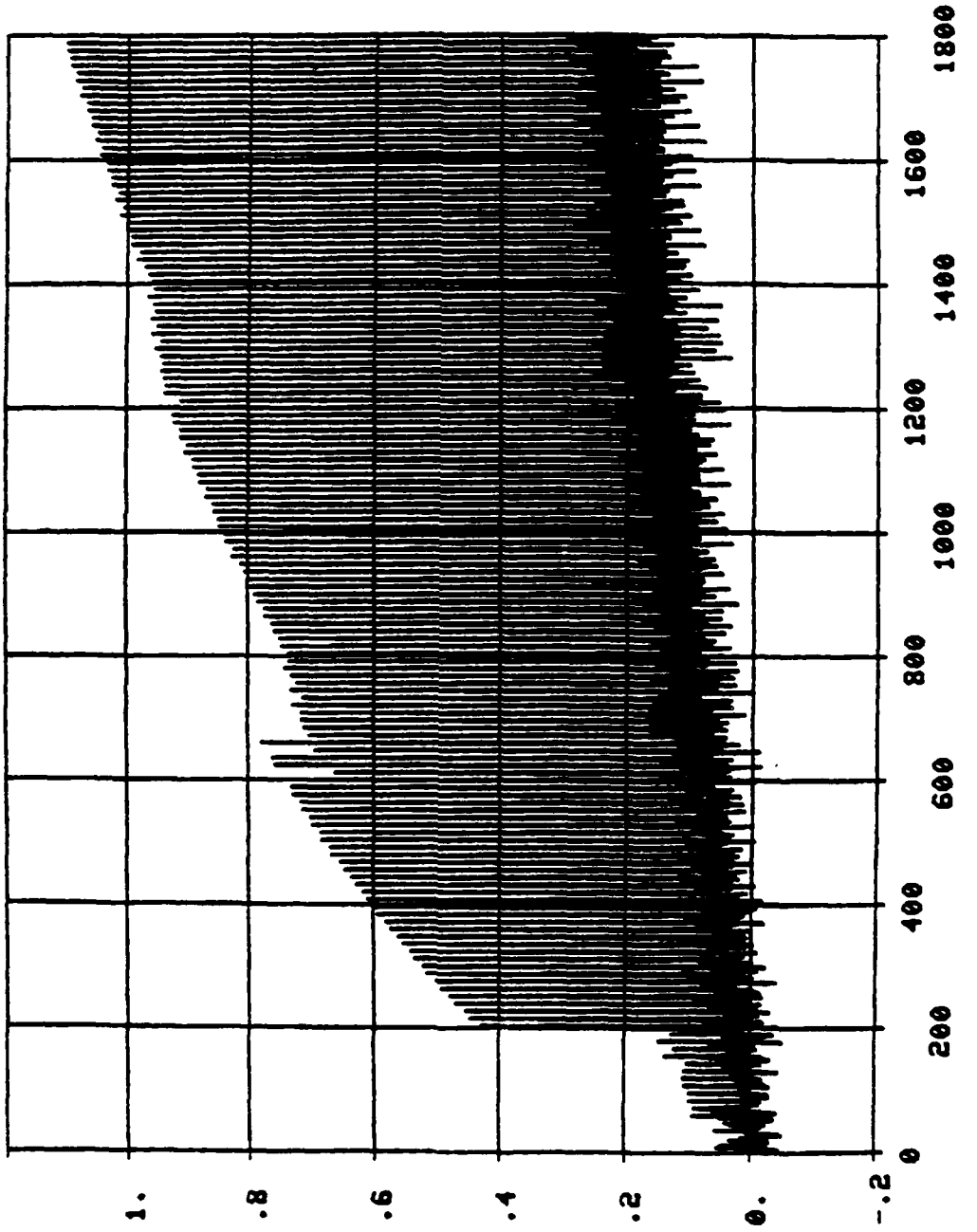

Figure 8. Tilt error obtained with 32 bit computation.

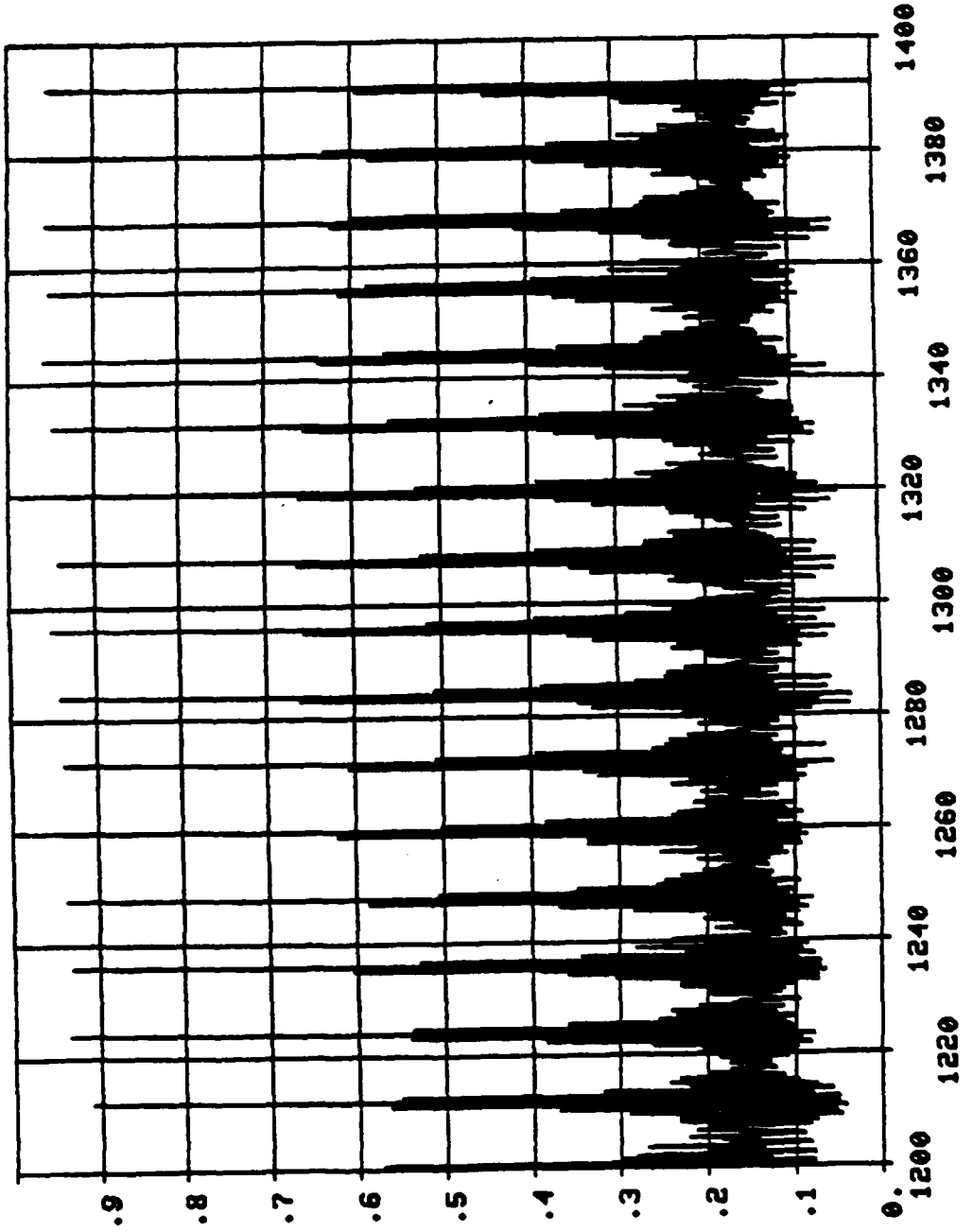

Figure 9. Tilt error obtained with 32 bit computation (expanded time scale).

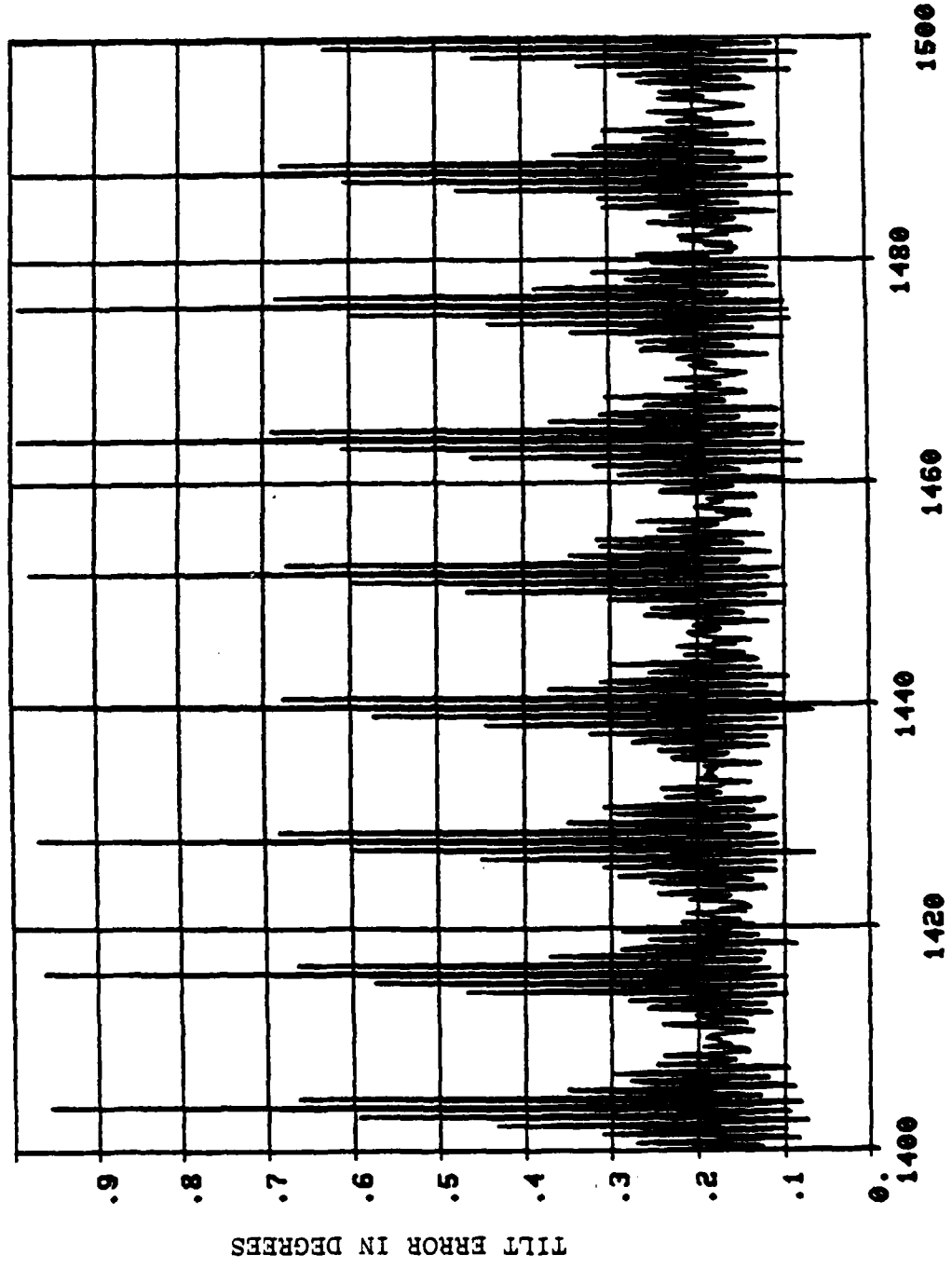

Figure 10. Tilt error obtained with 32 bit computation (expanded time scale).

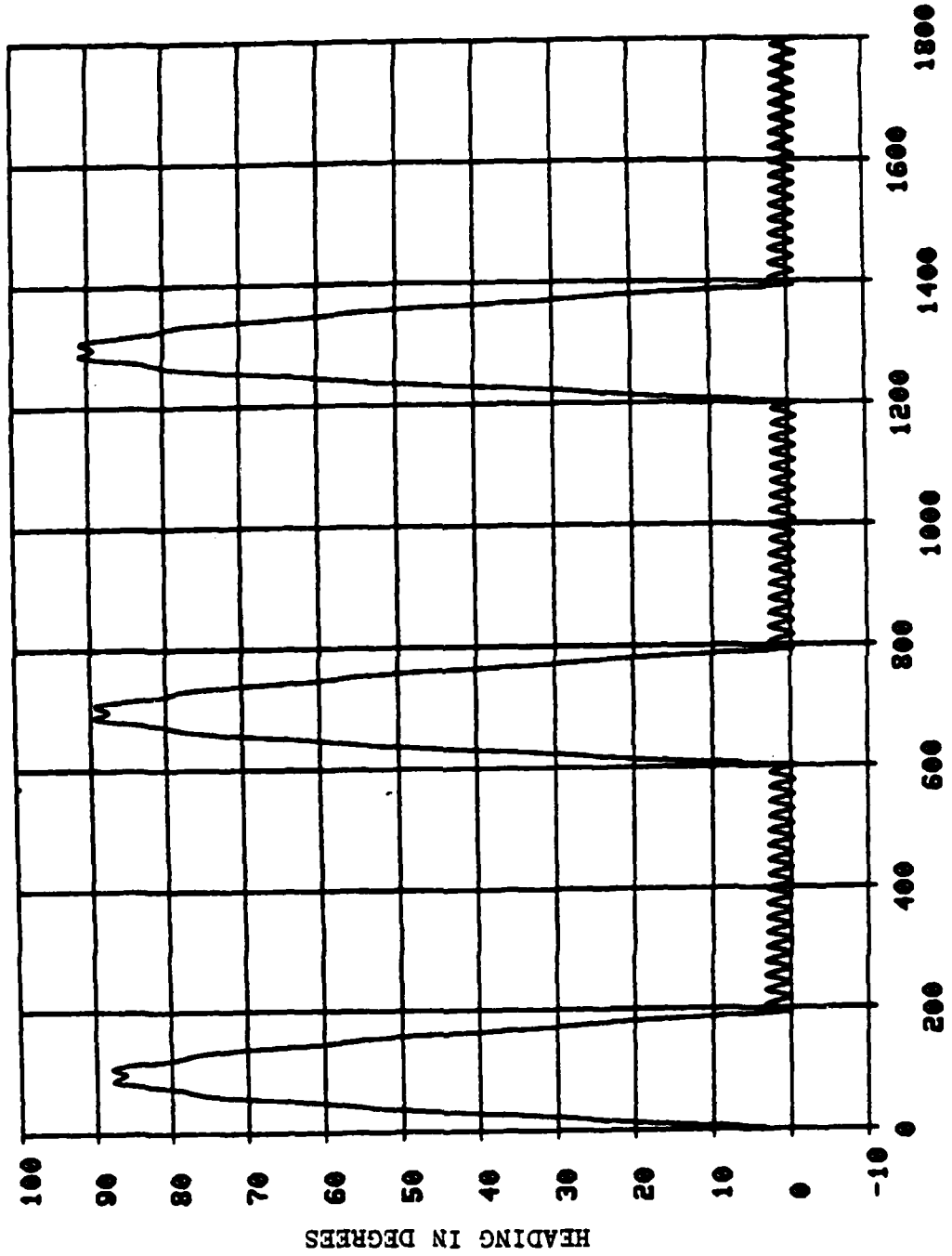

Figure 11. Heading obtained with 24 bit computation.

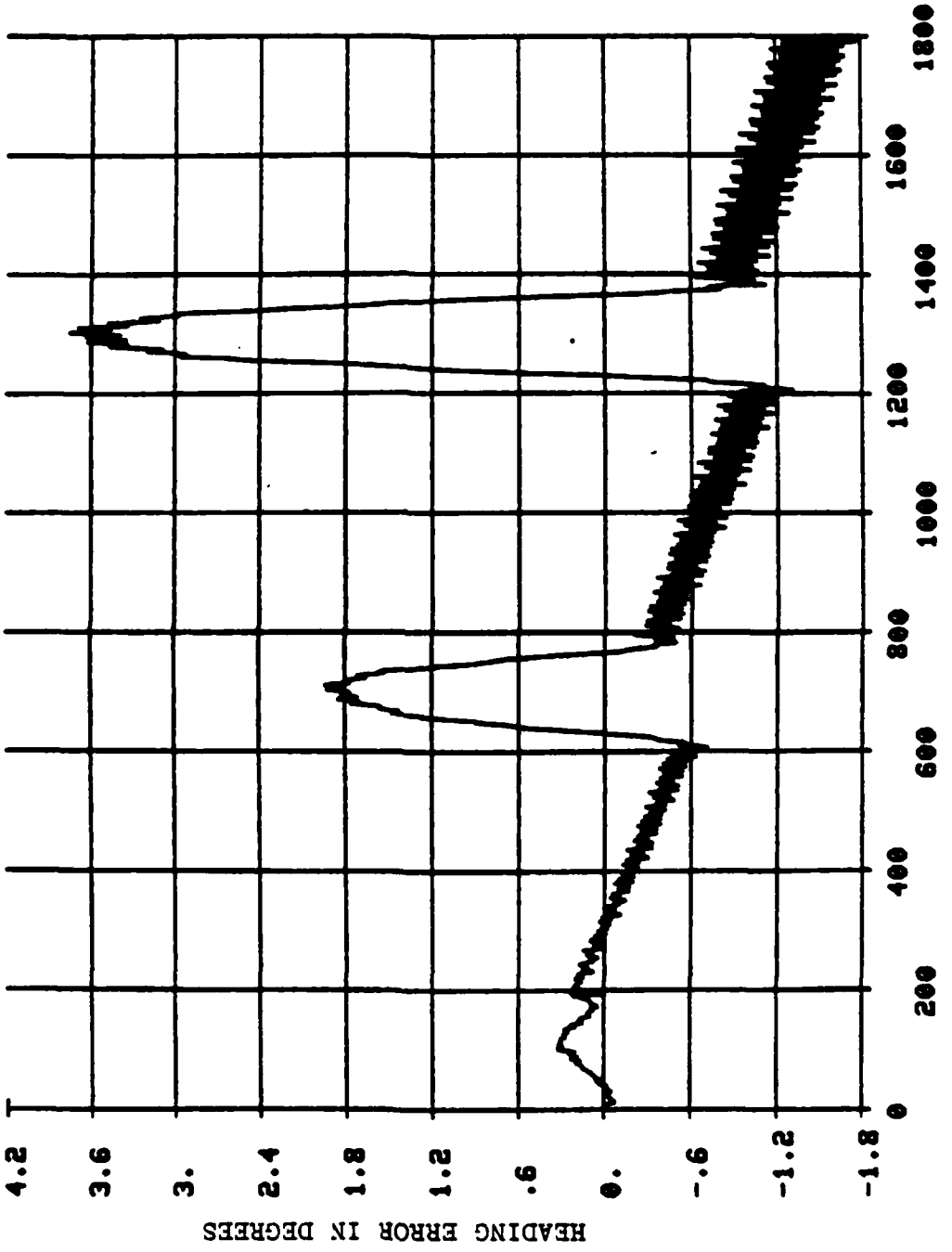

Figure 12. Heading error obtained with 24 bit computation.

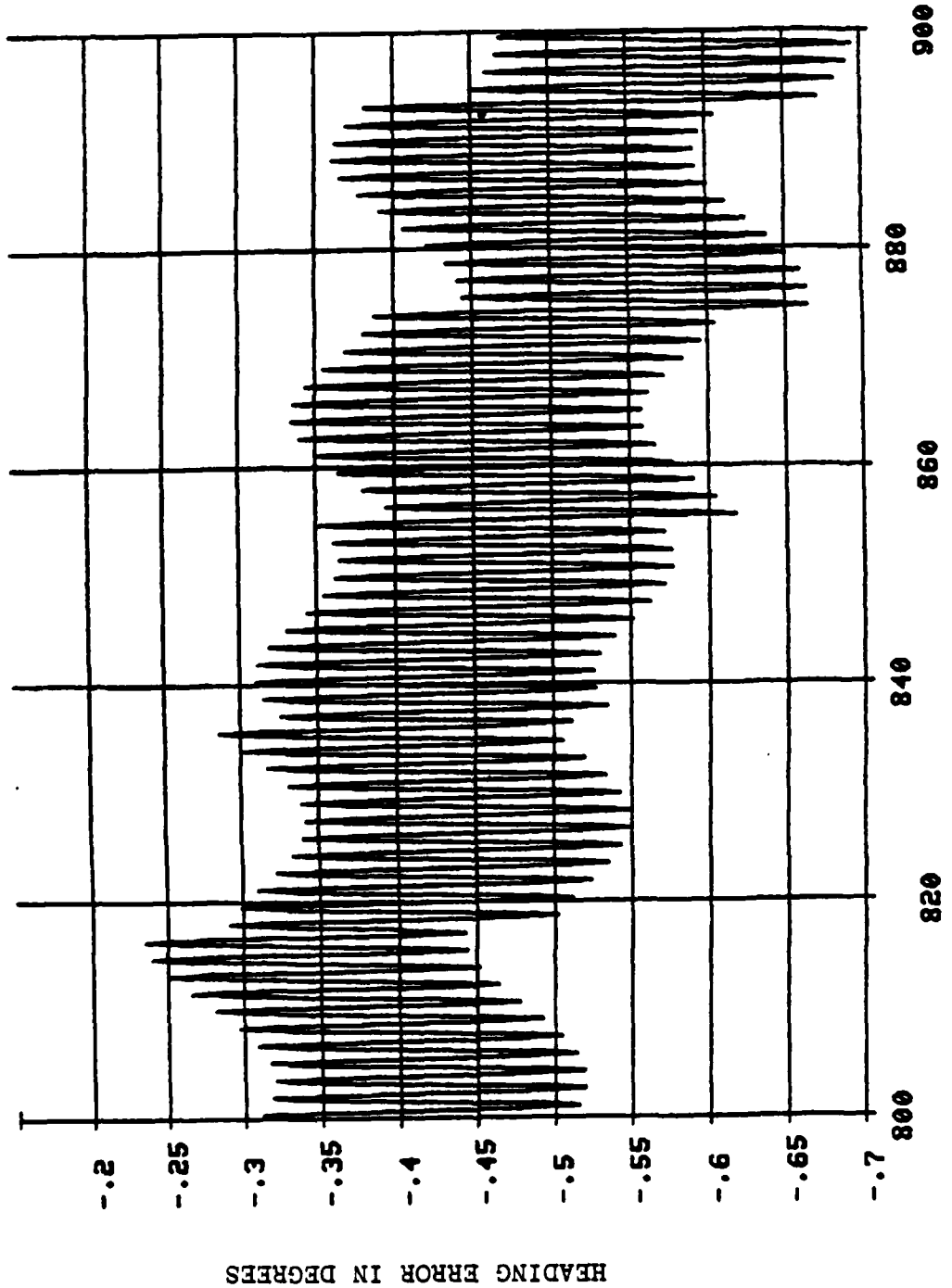

Figure 13. Heading error obtained with 24 bit computation (expanded time scale).

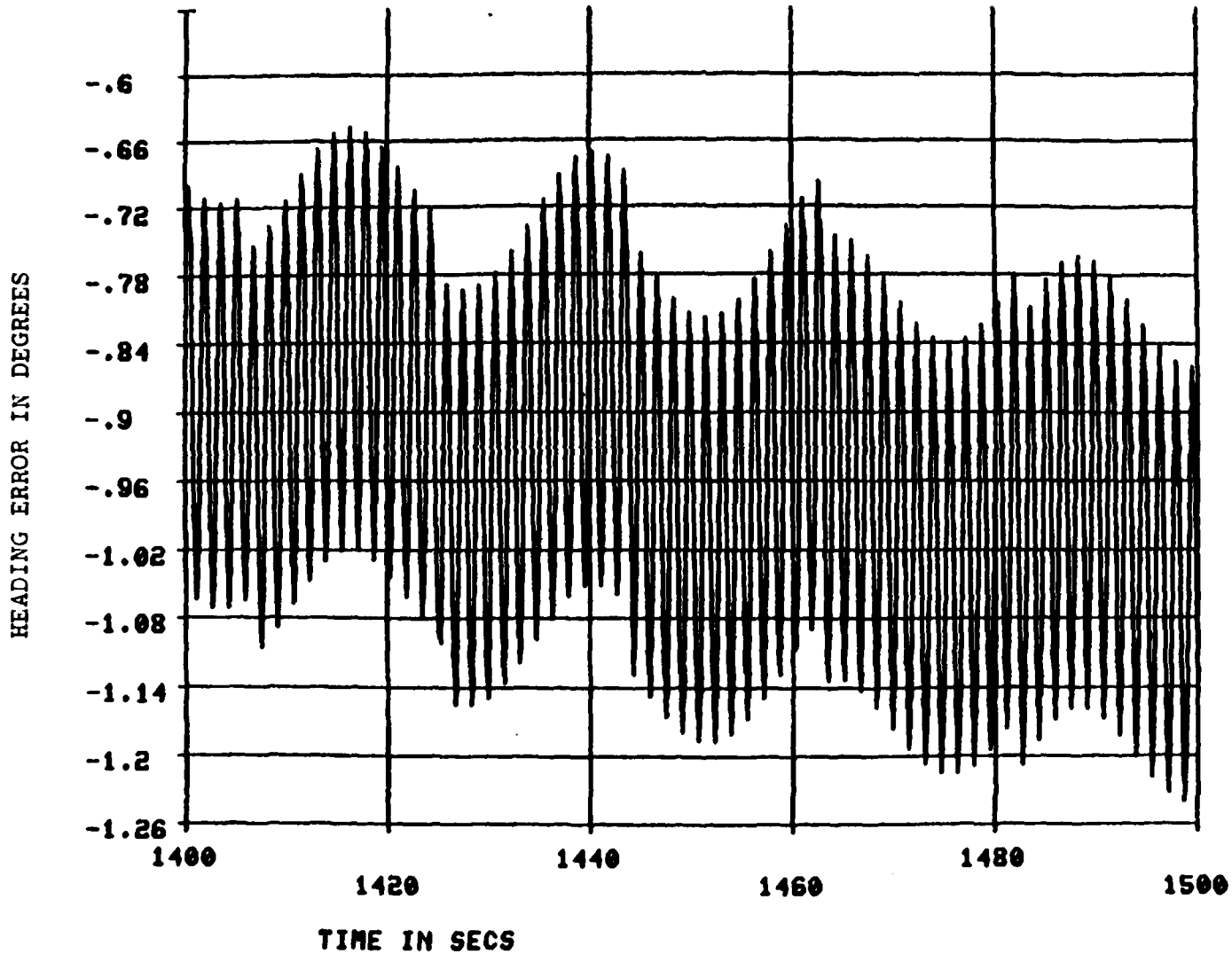

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Figure 14. Heading error obtained with 24 bit computation (expanded time schedule)

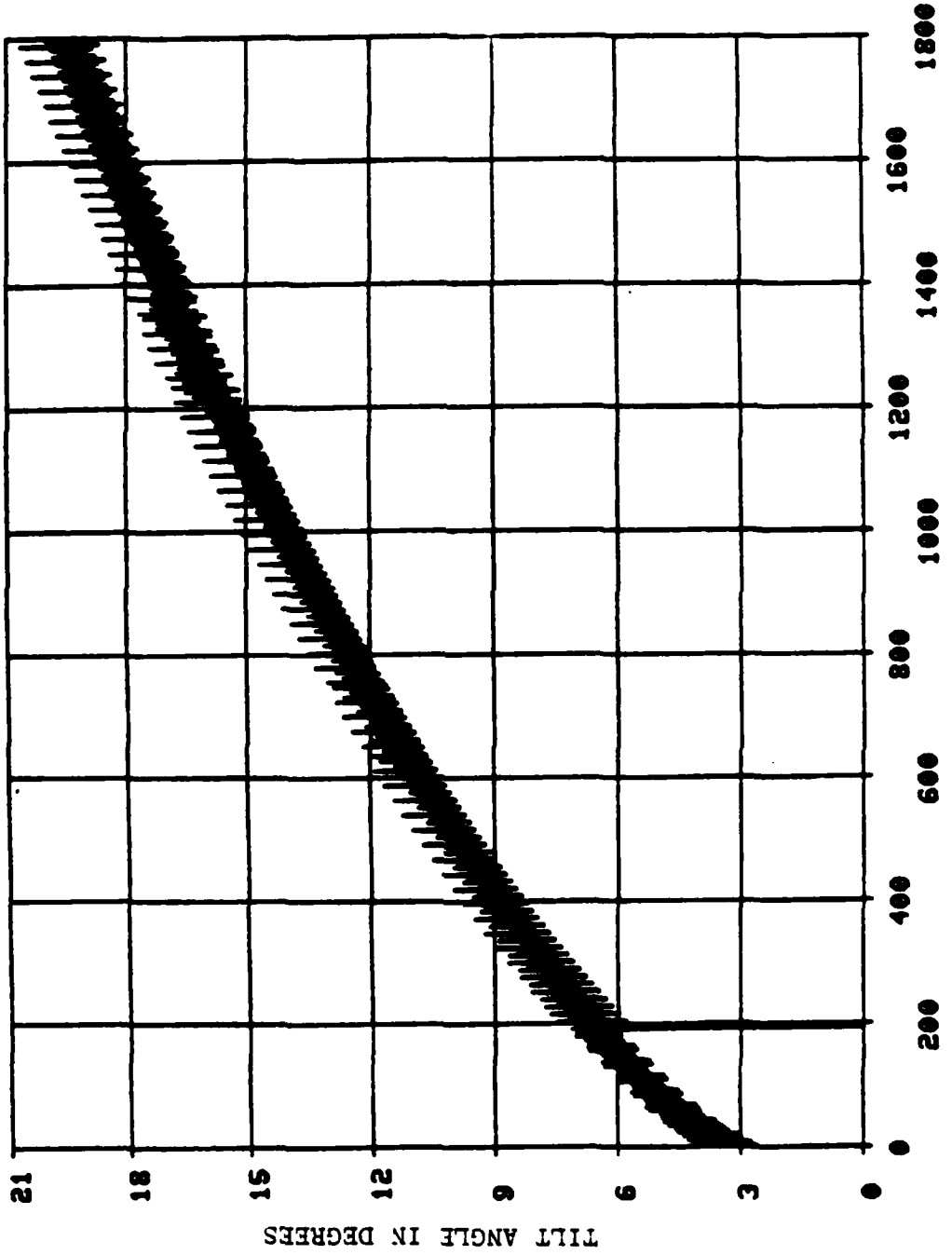

Figure 15. Tilt angle obtained with 24 bit computation.

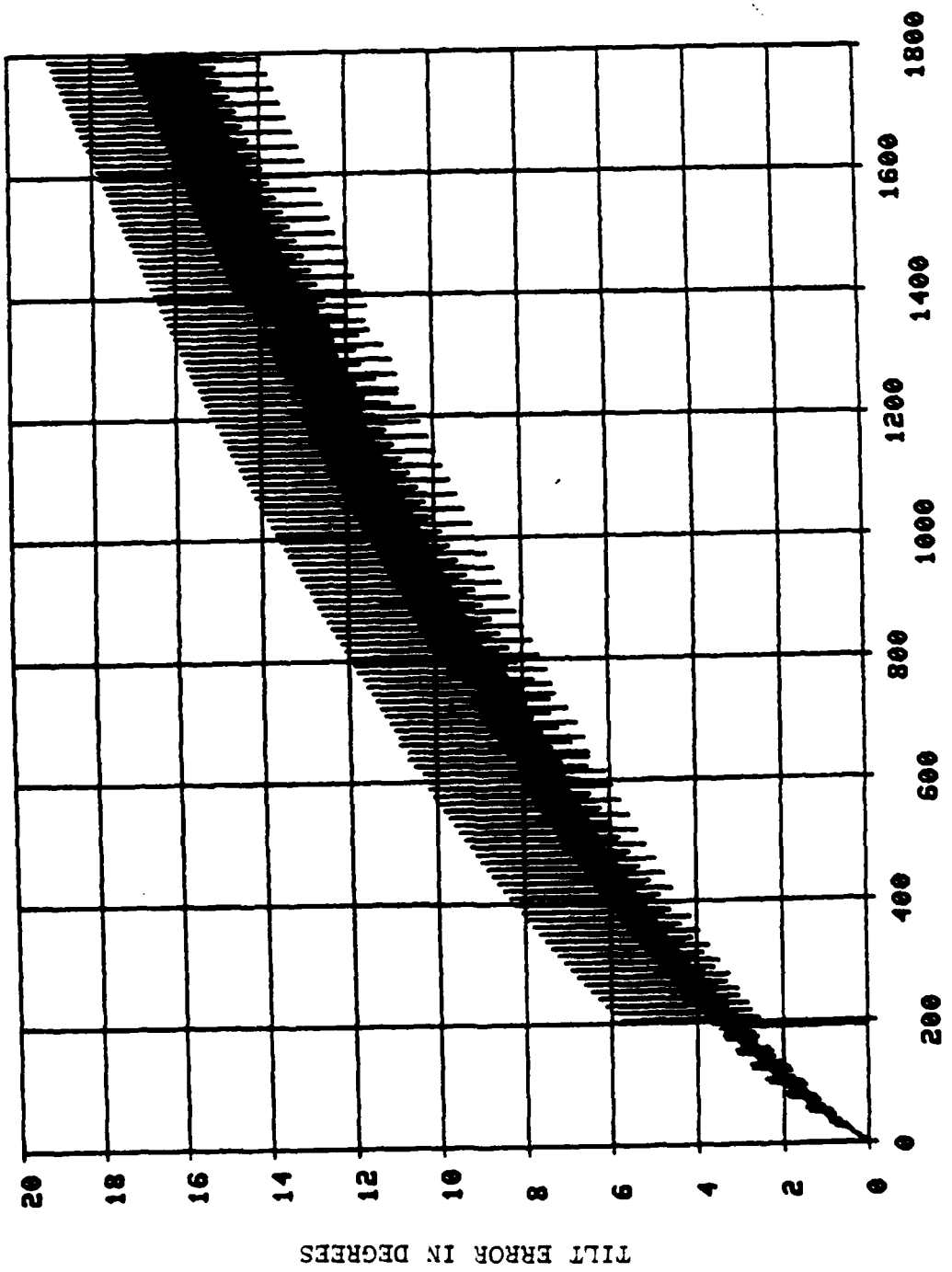

Figure 16. Tilt error obtained with 24 bit computation.

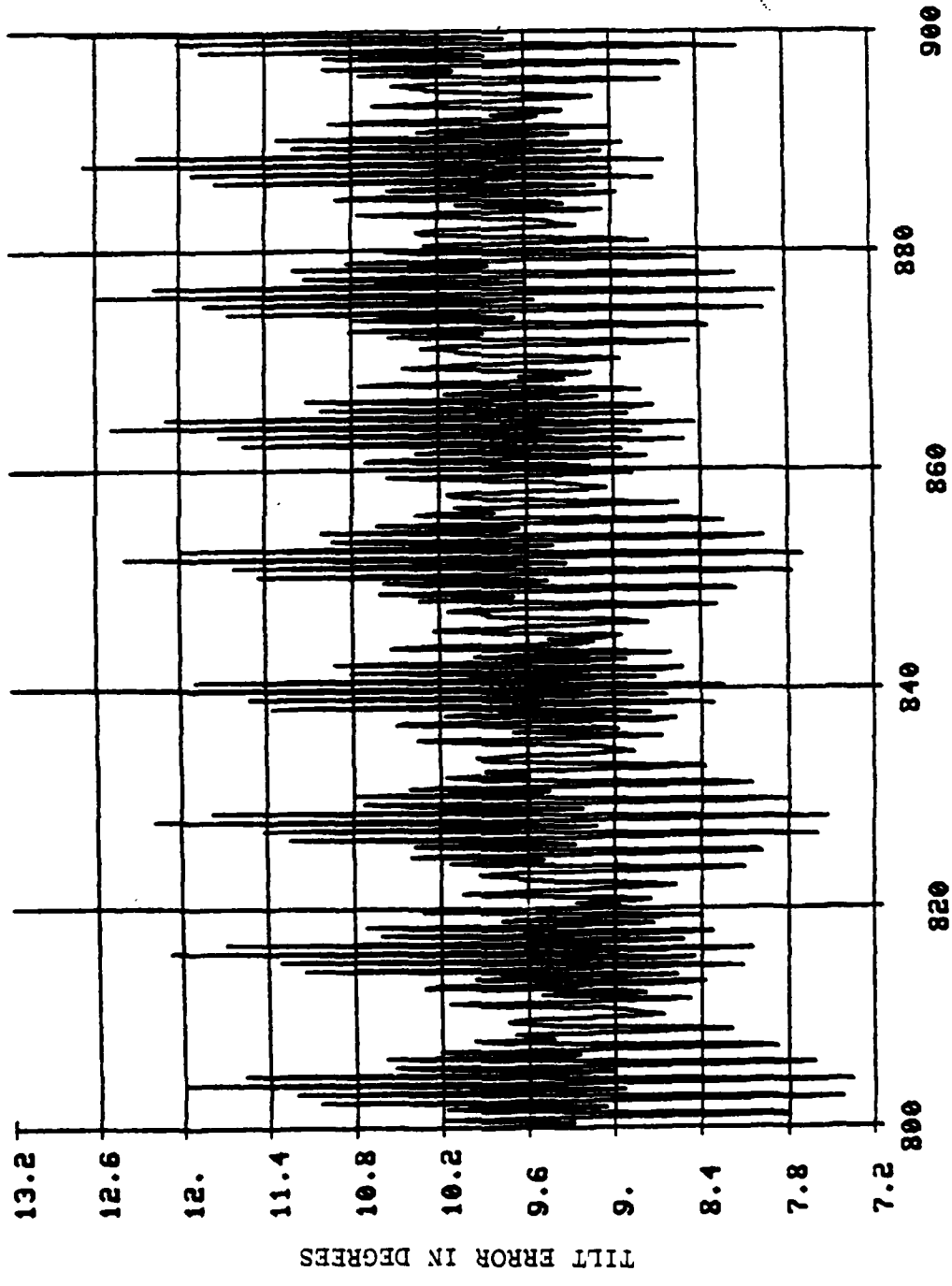

Figure 17. Tilt error obtained with 24 bit computation (expanded time scale).

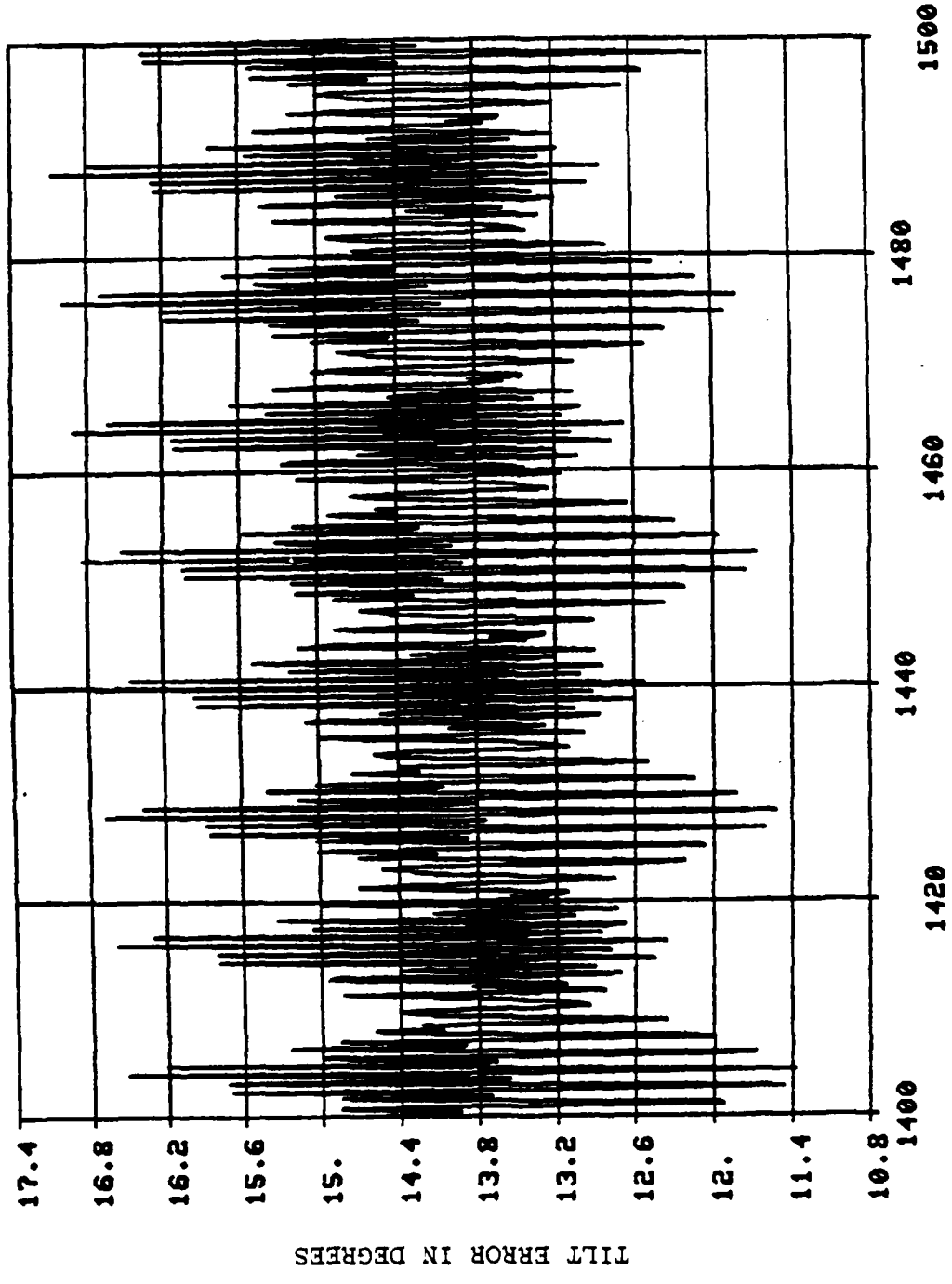

Figure 18. Tilt error obtained with 24 bit computation (expanded time scale).

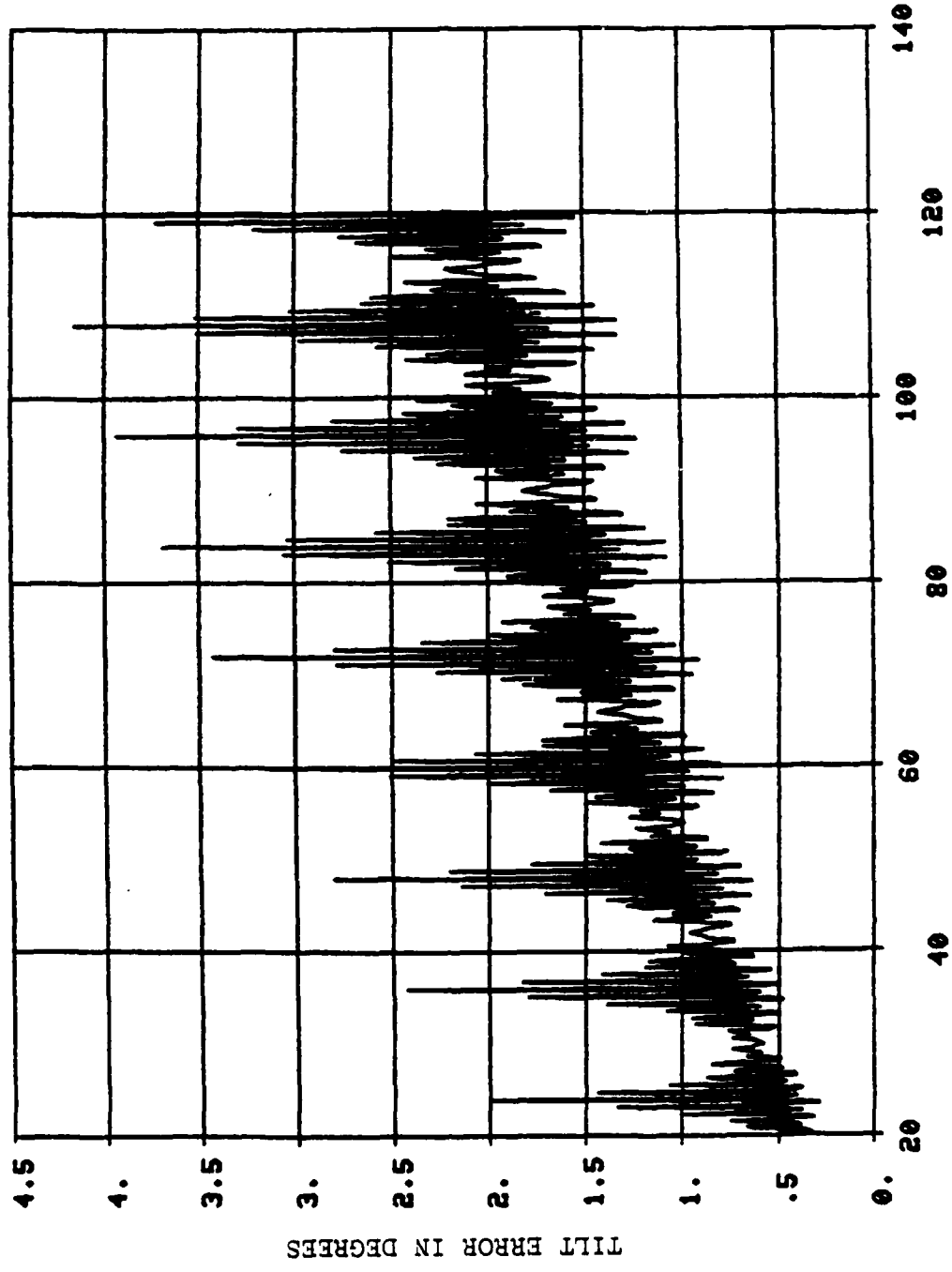

Figure 19. Tilt error obtained with 24 bit computation (expanded time scale).

4.3 Conclusions

The choice of 10 ms, second-order updating with 32-bit precision is demonstrated to be quite adequate. No significant long-term trend in heading error can be detected. The more sensitive tilt error shows a gradual growth to 0.2 deg after 30 min. The sine wave pattern of heading excursions during periods of no yaw motion as seen in Figure 3 is a result of the coning phenomenon (see Appendix D). The direction and rate of change of heading continuously change as the pitch and roll phase relationship varies.

A reduction to 24 bits shows a significant deterioration. In addition to no longer accurately following the 90 deg yaw excursions, a steady error accumulation to 1.5 deg at 30 min is apparent. The tilt error is a full 16 deg at 30 min.

A further comparison may be made of the determinants for the two cases. For the 32-bit case, the determinant of the attitude matrix at the end of 30 min was 0.99935. For the 24-bit case, the determinant equalled 0.8529, a considerable departure from the ideal unity.

Ten minutes worth of data was also obtained from a test of 32-bit, first-order processing. While no serious heading errors had yet developed, the determinant at the end of 10 min was a horrendous 1.45. It is safe to conclude that 10 ms. first-order processing is unsatisfactory.

Computer program listings for Section 4 are given in Appendix H.

5. MEASUREMENTS OF SENSOR CHARACTERISTICS

In addition to the studies of a heading sensor system which would incorporate a fluidics angular rate sensor, a concurrent measurement effort was undertaken with joint participation of APL and HDL personnel. These measurements were aimed at key fluidics sensor characteristics.

The tests were performed under admittedly benign laboratory conditions, but demonstration of feasibility of the existing sensor design was the immediate objective. As noted elsewhere in this report, the successful development of a heading sensor with a 1 deg/hr drift requires extremely accurate measurement of angular rate inputs. Whether or not the angular rate sensor will meet these objectives must await the construction of a combined sensor-transducer package which can then be subjected to precision testing under a variety of environmental conditions.

Measurement of angular rate was made via three stages of fluidic amplification and a Barocel pressure transducer. Data were recorded on an X-Y plotter. Angular rates were obtained with a rate table. Standard laboratory air was used as the power source with different venting arrangements. (A single output port reduces the ambient noise effects.)

Pertinent data from one set of tests are reproduced as Figure 20. An operating point of 13 mm Hg was selected as being least sensitive to supply pressure fluctuations.

The transfer function of output pressure change versus angular rate was 0.05 mm Hg per deg/s. The response to angular rate excursions of ±0.02 deg/s is shown on Plot 2 of Figure 20. Plot 3 shows a very discernible response to ±0.002 deg/s. Thus, the equivalent of earth rate (0.004 deg/s) can be seen by the sensor. To achieve the desired goal of 0.8 deg/hr, a sensitivity of 0.0002 deg/s is required. Since it is expected that rate signals will drive an integrator, it is the long-time integral which must be sensitive to rates of 0.0002 deg/s. Hence, even if such signal levels at the X-Y plotter would appear to be submerged in noise, conclusions about sensitivity for this application are premature unless suitable instrumentation is employed.

Plot 4 shows one measurement of a second key characteristic; namely, offset drift. The offset appears to have changed by the equivalent of a rate change of 0.001 deg/s over 24 min or a drift rate of 9 deg/hr/hr. In a simple

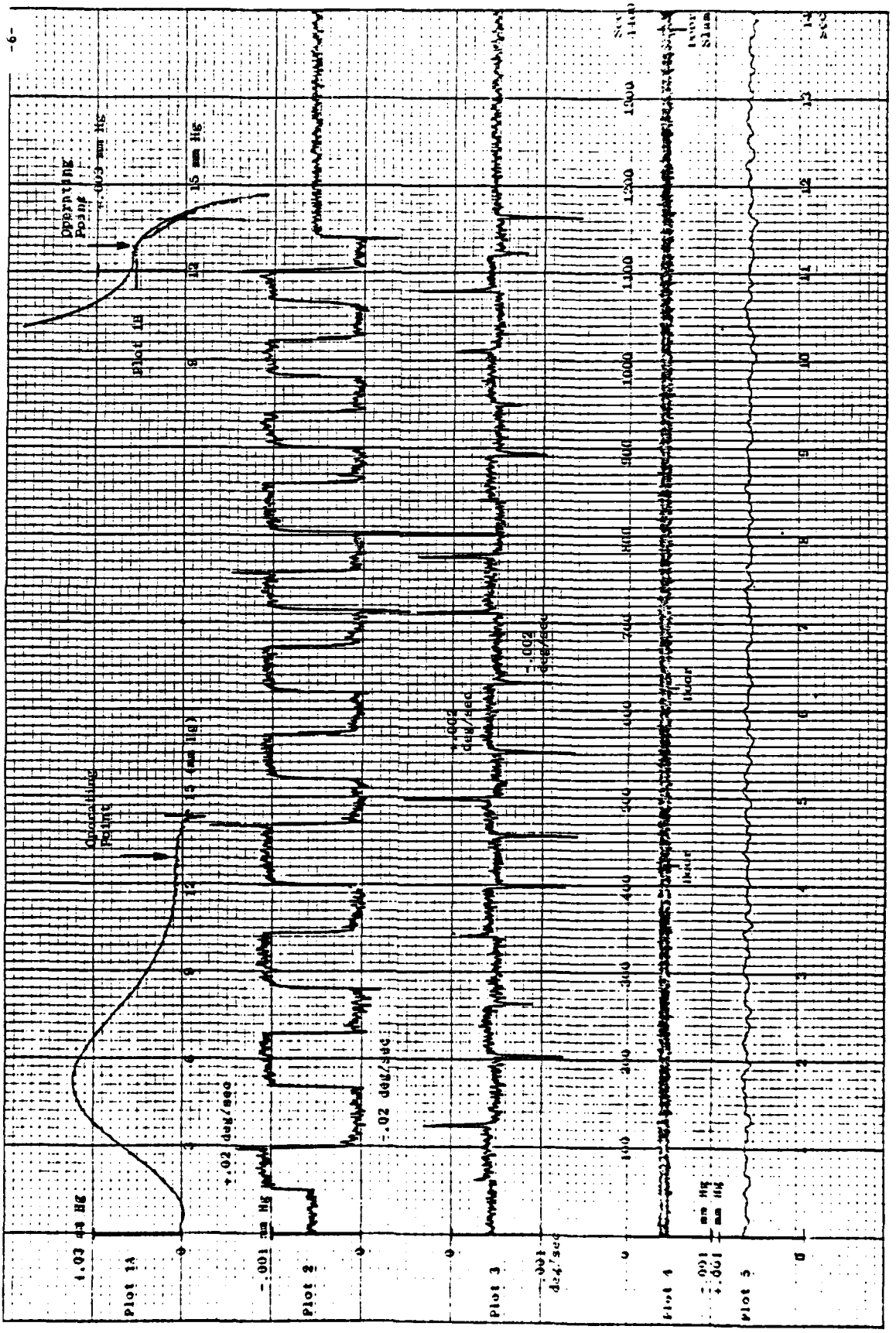

Figure 20. Sensor test results.

single-axis system, the angle is simply the integral of the rate. Hence, the error would be 4.5 deg after 1 hr and 40.5 deg after 3 hr. If, in a final system, constant drift rates are experienced and can be accurately determined during an alignment period, suitable compensation can be obtained within the processor. If offset changes cannot be predicted, their magnitudes would have to be further reduced.

It should be noted that an available sensor was used as the basis for these tests. A potential for improvement exists by optimizing the sensor design for the specific application at hand. (For example, the present high angular rate capability may be relaxed.)

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6. CONTRIBUTIONS TO SENSOR ERRORS

Results of the computer simulations given in Section 3 demonstrate that the limitations on rate sensor errors can be estimated by considering only a single-axis system. Hence, for a 1 deg/hr heading drift, the angular sensor should have an unpredictable (and hence uncompensated) bias of approximately the same magnitude or less. A general requirement may be stated as follows: the total effect of all sensor deficiencies should produce an output which, on the average, tracks the input angular rate with an error no greater than the desired heading drift. Many of the contributory factors will be scenario dependent. The "bias" of Section 3 should be recognized as the effective sum of many error sources.

6.1 Offset

Any unpredictable or unexpected shift of the null point is, of course, a bias in the sense of Section 3. This offset, in turn, may be the result of uncompensated temperature or pressure changes within the angular rate sensor or transducer.

6.2 Asymmetry

In Section 3, it was shown that scale factor errors of one percent or so would be tolerable. However, should there be a lack of symmetry of the transfer function, rectification of input motion results in a bias, as shown in Figure 21.

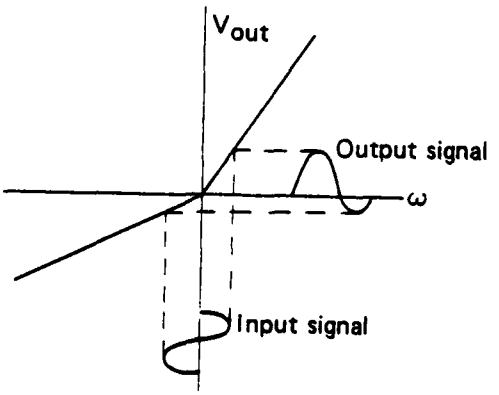

Figure 21 Nonsymmetrical transfer function.

If the input motion is sinusoidal with a 6 deg peak-to-peak amplitude, then 6 deg represents the integral of a half sine wave of the corresponding angular rate waveforms. If positive and negative scale factors differ by 1%, then a 0.06 deg error is generated every cycle. For motion with a 1.5 s period, the equivalent "bias" is 145 deg/hr. Therefore, considerably better matching of positive and negative characteristics (of the order of 0.02%) is required (after compensation).

6.3 Nonlinearity

A similar rectification effect can be produced from a nonlinear transfer function, even if symmetry is maintained, because of nonsymmetrical input motion.

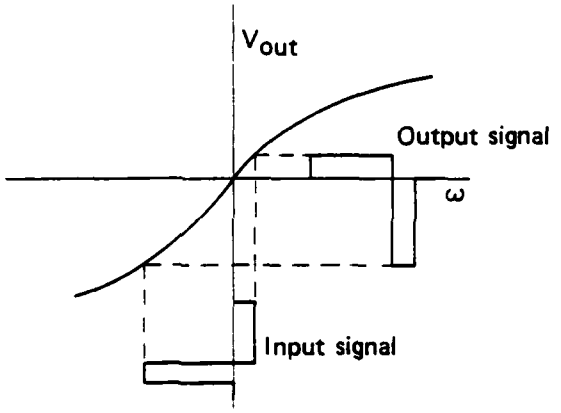

Figure 22 Nonlinear transfer function.

One can readily visualize a vehicle drifting slowly in yaw which is periodically corrected by the driver as illustrated in Figure 22. The drift is long and slow; the correction short and fast. For the vehicle to maintain its average direction, the integration of the input waveform must equal zero. But the integral of the output waveform delivers a "bias" to the computer by virtue of the unequal positive and negative scale factors. If a 5 deg drift were corrected every five seconds, a nonlinearity of 1% gives an angular error of 0.05 deg every five seconds or an equivalent "bias" of 36 deg/hr.

It will be noted that, insofar as certain characteristics such as non-linearity, hysteresis, g-sensitivity are concerned, the specification for an acceptable sensor is intimately related to the profile of vehicle motion. For the sensor to operate acceptably under all possible conditions, the angular rate error would have to be limited to the desired

drift rate independent of input motion. Hence, for a 1 deg/hr sensor ($\approx$ 0.0002 deg/s), and a maximum rate of 40 deg/s, the nonlinearity must be limited to 0.0005% of full scale. Since maximum rate is not encountered frequently, such a requirement is much too stringent. (The maximum angular rate corresponding to the example of 3 deg, 2.5 Hz. motion is 12 deg/s). The simple examples given here to illustrate the effects of asymmetry and nonlinearity indicate that 0.01% would be more realistic.

6.4 Hysteresis

Any memory exhibited by the device can result in an effective bias, again depending on the vehicle scenario. Ideally, the output of the sensor as a function of past history should vary by less than the desired drift rate.

6.5 Cross-Axis Sensitivity

Obviously, the sensor should not produce outputs for angular motion about any but its designated input axis. To the extent that this effect can be measured and is repeatable, compensation can be introduced into the processor.

6.6 Sensitivity to Linear Acceleration

If the sensor is g-sensitive, errors due to gravity can be compensated for since the attitude is always known in the computer (not without penalty, however; such additional computations will take additional time). Without accelerometers as part of the system, there would be no way to compensate for errors due to vehicle acceleration. A detailed computer run of the type of Section 3 could be made for a vehicle scenario of three-axis acceleration.

6.7 Two-Axis Quadrature Vibration

If two axes of the vehicle undergo quadrature motion, the third axis will show an effective rotation which is not sensed by the third axis sensor. This is the coning effect which must be properly handled by the computational procedures. Now, should the two axes experience quadrature vibration which is not true vehicle motion, the computer will erroneously calculate a third-axis rotation which never takes place. This vibration must be slow enough to be seen at the sampling intervals ($\approx$ 10 msec).

6.8 Rectification of Vibration

Another vibration effect occurs when vibrational motion is accompanied by vibrational tilt, resulting in a rectification effect.

The effects of vibration may be reduced via shock-mounting of an integrated 3-axis sensor package. The attitude of the package rather than the vehicle is tracked, allowing momentary insignificant vehicle heading errors with no error accumulation. Small angular differences between the sensor package and the vehicle would be of some concern if an attempt were made to combine a vehicle speedometer with the heading sensor to obtain position.

ACKNOWLEDGMENT

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R. L. Hickerson
V. F. Neradka
T. M. Rankin
J. G. Wall
L. L. Warnke

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3. W. F. Ball, "Strapdown Analysis, Part 4, Gyro Modelling and Gyro Storage Error," Naval Ordnance Test Station Technical Note 404-38, 15 June 1966.

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5. Goodman and Robinson, "Effects of Finitate Rotations on Gyroscopic Devices," Journal of Applied Mechanics, June 1958, pp. 210-213.

APPENDIX A.--COMPUTATIONAL ALGORITHM

This section briefly outlines the computational algorithm needed to continuously track a vehicle heading. No attempt was made in this study to determine an optimum approach. Rather, the purpose of this phase of the study was to demonstrate feasibility. Accordingly, the popular direction-cosine matrix technique was used, neglecting other possibilities (such as the use of quaternions) described in the literature[1,2]. With such a feasibility demonstrated, development efforts could be concentrated upon the input sensor and transducer. Feasibility consists of demonstrating acceptably small errors when the algorithms are limited to the capabilities of a modern microprocessor. Any marginal speed problems can be safely neglected since it is a small matter to add a hardware multiplier.

The direction cosine matrix is a nine element matrix which fully describes the attitude of the vehicle with respect to the local (or navigation) frame. Each element is the cosine of the angle between a pair of axes.

		Vehicle Axes		
		X	Y	Z
NAV Axes	X'	C_{11}	C_{12}	C_{13}
	Y'	C_{21}	C_{22}	C_{23}
	Z'	C_{31}	C_{32}	C_{33}

X, Y, and Z are the longitudinal, transverse, and vertical axes of the vehicle. For a right-hand system, positive directions are taken as forward, right, and down. X', Y', Z' are the corresponding local frame axes and are chosen to coincide with north, east, and down*, respectively. Any vector

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[1] J. C. Wilcox, "A New Algorithm for Strapped-Down Inertial Navigation," IEEE Transactions on Aerospace and Electronics Systems, Vol. AES-3, No. 5, September 1967, pp. 796-802.

[2] J. E. Bortz, "A New Mathematical Formulation for Strapdown Inertial Navigation," IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-7, No. 1, January 1971, pp. 61-66.

*Toward the center of the earth.

in the vehicle frame may be transformed to a vector in the NAV frame by premultiplying by this direction cosine matrix (C_V^N).

So defined, the elements C_{11} and C_{21} give the north and east components, respectively, of the vehicle's longitudinal axis. Therefore, heading may be extracted from the C-matrix by computing the arc tangent of C_{21}/C_{11}. The more difficult task is the accurate updating of the C-matrix as angular increments over a sample period are developed from the angular rate sensors.

The rate of change of the C-matrix is given by

$$\dot{C} = CW, \tag{1}$$

where W is a matrix describing the angular rates of the vehicle's axes, $\dot{x}$, $\dot{y}$, $\dot{z}$:

$$W = \begin{pmatrix} 0 & -\dot{z} & \dot{y} \\ \dot{z} & 0 & -\dot{x} \\ -\dot{y} & \dot{x} & 0 \end{pmatrix} . \tag{2}$$

For <u>constant velocity</u>, the solution for C at any time t is

$$C(t) = C(o)e^{Wt}$$

or, in terms of samples every n^{th} instant,

$$C_{n+1} = C_n \, e^{W\Delta t} . \tag{3}$$

The matrix $W\Delta t$ is now a matrix of angular increments rather than angular rates.

The exponential term may be expressed as an infinite series, leading to

$$C_{n+1} = C_n \left[I + W\Delta t + \frac{(W\Delta t)^2}{2} + \ldots\ldots \right] , \qquad (4)$$

where I is the identity matrix.

The microprocessor is thus required to update the most recent C-matrix after each sampling, with summation and matrix multiplication as indicated by equation (4). If only the terms I and $W\Delta t$ are used, the process is simple first-order updating. Greater accuracy is obtained by including the term $\frac{(W\Delta t)^2}{2}$, referred to as second-order updating.

Prior to updating per equation (4), a correction must be applied to $W\Delta t$ to reflect the effect of earth rate. This is necessary since the angular rate sensors measure the vehicle's motion with respect to inertial space, whereas the C-matrix gives the vehicle's attitude with respect to the local frame which, in turn, is rotating. For the earth spin rate Ω and latitude L, the NAV (local) frame components are Ω cos L, 0, and $-\Omega$ sin L. Premultiplying these components by the transpose of the C_V^N matrix then gives the earth rate effect as seen by the vehicle axes. Subtracting from the measured angular rates will give the net rates (or increments) with respect to the NAV frame.

Some 42 multiplications per sample are indicated for the foregoing procedures. Assuming 200 μsec per 32-bit multiplication, a 10 msec sampling period should allow sufficient time for computation. A hardware multiplier can always be used for faster execution, if necessary, at a small (< $200) additional cost.

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APPENDIX B.--ALTERNATE ALGORITHMS

The computational procedure described in Appendix A is fairly conventional for "strap-down" inertial systems. When so desired, ground speed information may be combined with attitude to calculate position. If display of heading only is the ultimate objective, then computation of heading may be performed relatively inaccurately and infrequently. However, the underlying attitude matrix must be updated with precision. A tradeoff may be implemented wherein the chore of matrix updating is simplified at the expense of a more complex heading calculation.

The C-matrix is now chosen to describe the vehicle attitude with respect to inertial space, C_V^I. Updating of this matrix from the measured angular increments requires no prior correction for earth rate, thus eliminating the multiplication involving the transpose of C_V^N. Only when a heading calculation is to be performed is an additional coordinate rotation required.

In one version of this approach, the inertial axes are selected to coincide with the local navigation frame at the start of a mission, simplifying the initialization procedure. Accurate updating of the C_V^I matrix from the angular increment readouts proceeds every sampling instant. When at any time t it is desired to compute heading, it is first necessary to compute C_I^N (inertial-to-navigational) matrix at that time. The change of attitude of the navigational frame in inertial space over the time t may be visualized as a succession of three rotations:

(1) rotation +L deg about Y axis (pitch up)

(2) rotate $+\Omega t$ about new X axis

(3) rotate -L deg about new Y axis (pitch down)

(L is latitude and Ω is the earth spin rate.)

The product of the corresponding rotation matrices gives the matrix C_I^N. The elements C_{11}, C_{21}, and C_{31} of the C_V^I matrix

form a vector describing the direction of the vehicle's longitudinal axis with respect to the inertial frame. When premultiplying by C_I^N, the components with respect to the navigational frame are obtained, from which heading may be calculated in the usual manner.

An even great simplification may be obtained at the cost of greater complexity in the initial alignment procedure. Earth-aligned inertial axes are chosen:

X - directed toward and perpendicular to the earth's axis.

Y - east

Z - parallel to negative spin axis

(X and Y lie in a plane parallel to the equatorial plane.)

Only two rotations are needed to transform from this inertial frame to the local NAV frame:

(1) rotate $-\Omega t$ about the Z axis

(2) rotate (90 deg-L) about the new Y axis

The calculation of heading proceeds as before.

These procedures have merit only if heading is to be calculated infrequently. If position navigation is required, continuous updating of heading may be needed to minimize errors, in which case the use of inertial frames would be of no benefit.

APPENDIX C.--"PRECISION" MATRIX UPDATING USED IN SECTION 3, BODY OF REPORT

The computer program described in Section 3 attempts to eliminate computational errors so that the heading errors obtained are due solely to angular rate sensor errors. One effect of the quantizing of input motion is to allow for a more time-consuming but highly precise updating.

Given the three angular increments between sampling instants, the microprocessor-based algorithm is described by equation (4), Appendix A:

$$C_{n+1} = C_n \left[I + W\ t + \frac{(W\Delta t)^2}{2} + \ldots . \right]$$

With a large-scale computer available, a closed-form solution may be utilized for the above infinite series:

$$C_{n+1} = C_n\ (\phi) \quad ,$$

where the elements of ϕ are

$$\phi_{11} = \frac{x^2}{\beta^2} + \frac{\Delta y^2 + \Delta z^2}{\beta^2} \cos \beta$$

$$\phi_{12} = \frac{-\Delta z}{\beta} \sin \beta + \frac{\Delta x\ \Delta y}{\beta^2} (1 - \cos \beta)$$

$$\phi_{13} = \frac{\Delta y}{\beta} \sin \beta + \frac{\Delta x\ \Delta z}{\beta^2} (1 - \cos \beta)$$

$$\phi_{21} = \frac{\Delta z}{\beta} \sin \beta + \frac{\Delta x\ \Delta y}{\beta^2} (1 - \cos \beta)$$

$$\phi_{22} = \frac{\Delta y^2}{\beta^2} + \frac{\Delta z^2 + \Delta x^2}{\beta^2} \cos \beta$$

$$\phi_{23} = -\frac{\Delta x}{\beta} \sin \beta + \frac{\Delta y\ \Delta z}{\beta^2} (1 - \cos \beta)$$

$$\phi_{31} = \frac{-\Delta y}{\beta} + \frac{\Delta x\ \Delta z}{\beta^2} (1 - \cos \beta)$$

$$\phi_{32} = \frac{\Delta x}{\beta} \sin \beta + \frac{\Delta y\ \Delta z}{\beta^2} (1 - \cos \beta)$$

$$\phi_{33} = \frac{\Delta z^2}{\beta^2} + \frac{\Delta y^2 + \Delta x^2}{\beta^2} \cos \beta$$

$$\beta^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 \ .$$

A more convenient, equivalent form is obtained for the computer updating via the following steps[1]:

$$\beta^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$$

$$\beta = \sqrt{\beta^2}$$

$$S = \frac{\sin \beta}{\beta}$$

$$Q = \frac{1 - \cos \beta}{\beta^2}$$

$$\phi_{11} = 1 - Q(\Delta y^2 + \Delta z^2)$$

$$\phi_{12} = Q\ \Delta x\ \Delta z - S\Delta z$$

$$\phi_{13} = Q\ \Delta x\ \Delta z + S\Delta y$$

---

[1] W. F. Ball, "Strapdown Analysis, Part 4, Gyro Modelling and Gyro Storage Error," Naval Ordnance Test Station Technical Note 404-38, 15 June 1966.

$$\phi_{21} = Q\,\Delta x\,\Delta y + S\Delta z$$

$$\phi_{22} = 1 - Q(\Delta x^2 + \Delta z^2)$$

$$\phi_{23} = Q\,\Delta x\,\Delta z - S\Delta x$$

$$\phi_{31} = Q\,\Delta x\,\Delta z - S\Delta y$$

$$\phi_{32} = Q\,\Delta y\,\Delta z + S\Delta x$$

$$\phi_{33} = 1 - Q(\Delta x^2 + \Delta y^2)$$

For each update to be performed, the ϕ matrix is developed from the three angular increments in a subroutine. The previous C-matrix is then multiplied by the ϕ matrix to obtain the updated C-matrix.

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APPENDIX D.--CONING MOTION AND ITS SIGNIFICANCE

This section discusses some aspects of coning motion with a view towards understanding its impact on attitude creep, computer simulation, and processing requirements.

The term "coning" derives from the fact that if two axes are subjected to sinusoidal motion of equal amplitude and frequency but 90 deg out of phase, the third axis describes a cone in space. In general, any combination of motion about two axes will result in changes of orientation about all three axes. The computer's function is to accurately track this changing attitude.

Sequential Pitch and Roll

Some insight may be obtained by examining the effect of sequential pitch and roll operations. For a rotation θ about any axis, the corresponding transformation matrix contains 1, cos θ, cos θ in the main diagonal. Sin θ and -sin θ appear once in symmetrical off-diagonal positions, with zero in the remaining positions. Rotation in the opposite direction ($-\theta$) results in the identical matrix except for a change of sign in the sine terms, equivalent to the transpose of the original matrix. Hence, if P is the rotation matrix for a pitch of θ and R is the rotation matrix for a roll of θ, the sequence of pitch $+\theta$, roll $+\theta$, roll $-\theta$, and pitch $-\theta$ is given by

$$P^T R^T R P = P^T I P = I \text{ (unity matrix)} ,$$

and we return to the starting position. This is motion where the Z axis retraces its path. If there is any misalignment in the C-matrix to begin with, its errors remain undisturbed.

If the Z axis does not retrace its path, such as in the sequence pitch $+\theta$, roll $+\theta$, pitch $-\theta$, roll $-\theta$, a net change in attitude occurs. (This should be intuitively obvious, since the positive and negative rotations are made about axes whose spatial orientation are different). The sequence of +pitch and +roll is given by

$$RP = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ \sin^2\theta & \cos\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & -\sin\theta & \cos^2\theta \end{pmatrix}$$

The sequence of -pitch and -roll is as above but with $-\theta$ substituted for $+\theta$. The resulting matrix is post multiplied by RP to obtain the final transformation matrix. When this is done, the following terms of interest result:

$$C_{11} = \cos^2\theta + \sin^2\theta \cos\theta$$

$$C_{12} = -\sin^2\theta$$

$$C_{33} = 2 \sin^2\theta \cos\theta + \cos^4\theta$$

For $\theta = 5$ deg, a change of heading $(\tan^{-1} \frac{C_{12}}{C_{11}})$ of 0.435 deg is obtained. A very small change of tilt $(\cos^{-1} C_{33})$ of 0.027 deg also results. Note that, in this case, if the C-matrix is misaligned to begin with, the computer will calculate net motion about an axis whose orientation is in error (even with perfect calculation). Hence, errors can grow as a result.

The motion about the third axis is real motion, the result of stipulated motion about the other axes. Much of the literature assumes, as a condition, that there is no cumulative motion about the third axis. Coning is then interpreted as that compensating axis rotation ($\dot{z}$ in this case) which would keep the heading constant.

Coning Theorem

A very useful theorem[1] states that if some motion brings an axis (say Z) back into coincidence with its starting position, a Z rotation will have been experienced equal to the solid angle swept out by the Z axis. This rotation in addition to any rotation due to the integral of $\dot{z}dt$. A simple demonstration is to allow a level vehicle to pitch up 90 deg, roll 90 deg, and pitch down 90 deg. The vehicle undergoes a heading change of 90 deg with no sensed Z axis rotation whatsoever. Applying the theorem, the Z axis returns to its original orientation after sweeping out 1/8 of a sphere or $\frac{4\pi}{8}$ radians.

For the coning motion test of Appendix G, the Z axis sweeps out a cone of half angle 36.85 deg five times, while the heading rotates 360 deg. The solid angle per sweep is $2\pi(1 - \cos 36.85 \text{ deg}) = 0.4\pi$. A full rotation of 2π therefore requires five such conical sweeps.

For the 5 deg pitch and roll example of this section, a 5 deg x 5 deg square is swept out. If the slight misalignment of the Z axis after one cycle is neglected, the approximate solid angle is $\frac{25}{3283} \times \frac{180}{\pi} = 0.435$ deg, in agreement with the previous calculation of heading change.

Significance for Computer Simulation

The "creep" of heading and tilt for the vehicle scenarios of Sections 3 and 4 may be understood as coning effects resulting from a stipulation of 3-axis motion. The input motion of Section 3 is due to irregular quantizing of the underlying sine wave inputs. This irregularity destroys the compensating effects of the scenarios of Section 4, leading to greater magnitudes of creep.

The received plots of pitch and roll for vehicle test runs show an in-phase relationship. Assuming that such a phase relationship is not universal and knowing that quadrature effects are more severe, some of the runs were changed to reflect pitch and roll inputs of slightly different frequencies. Hence, all phase relationships are experienced. The results indeed show greater perturbations for the latter inputs.

---

[1] Goodman and Robinson, "Effects of Finite Rotations on Gyroscopic Devices," Journal of Applied Mechanics, June 1958, pp. 210-213.

The plots of heading for the simulations of Section 4 are based on pure sine wave inputs, again with slightly differing pitch and roll frequencies. The effect on heading is clearly seen during the periods of no yaw (e.g., 200 to 600 sec.). As the phase between pitch and roll changes steadily, the heading drift changes direction, producing the beat pattern seen.

It must be emphasized that the foregoing effects describe real motion which must result if the 3-axis inputs are as specified. What is principally of concern is the ability of the system's output to properly define the attitude.

Significance for Sampling Interval

It has previously been mentioned that the computation algorithm is a "constant velocity" algorithm. To the extent that velocity is not constant between updates, some error will be introduced. It naturally follows that the maximum acceptable sampling interval is dependent on vehicle angular acceleration. If it is assumed that continuous coning motion due to pitch and roll represents a severe test, a simple intuitive estimation of drift may be made with the aid of the coning theorem.

Assume a worst case of quadrature motion. For equal amplitude pitch and roll, the Z-axis describes a cone, as shown in Figure D-1. The period of this sweep equals the pitch roll period. The half-cone angle equals the pitch and roll amplitude.

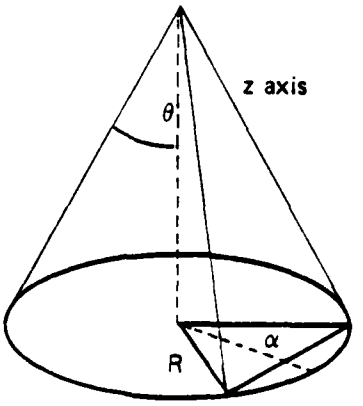

Figure D-1 Conical motion of Z-axis.

For small angles, there is little error in considering the spherical area intercepted by the cone to be a plane. The solid angle swept out per cycle is then proportional to the area of the plane circle (πR^2). The processor, however, assumes that the Z axis describes segments of the circle according to the sampling interval. It is assumed that the angular rate sensors are <u>not</u> sampled. Their outputs are continuously fed into integrators and it is the integrators which are periodically sampled. Hence, the angular position read at each sample is correct but the constant velocity algorithm assumes that the new position was attained via constant angular rate of the X and Y axes. The vehicle is thus thought to have rotated about a fixed axis in the X-Y plane. With no additional computation errors, a vehicle rotation about the Z axis will be calculated proportional to the area represented by the sum of the segments of the circle rather than the circle itself.

The area per segment is

$$A = R \sin \alpha \cdot R \cos \alpha = \frac{R^2}{2} \sin 2\alpha \ .$$

For N segments per circle,

$$\alpha = \frac{2\pi}{2N}$$

and the total area of N segments is

$$NA = \frac{NR^2}{2} \sin \frac{2\pi}{N} \approx \frac{NR^2}{2} \left[\frac{2\pi}{N} - 1/6 \left(\frac{2\pi}{N}\right)^3\right].$$

The second term within the brackets represents the departure from the area of the circle. The <u>fractional</u> error is

$$\text{Fractional error} = \left(\frac{2\pi}{N}\right)^3 \times 1/6 \times \frac{N}{2\pi} = 1/6 \left(\frac{2\pi}{N}\right)^2$$

Now, according to the coning theorem, the Z-axis rotation equals the solid angle swept out by the Z axis $\approx \pi\theta^2$ per cycle. The processor error is then

$$\varepsilon \approx \pi\theta^2 \times 1/6 \left(\frac{2\pi}{N}\right)^2 = 2/3\ \pi^3 \frac{\theta^2}{N^2} \text{ per cycle}$$

If T_p = pitch and roll period and T_s = sampling period:

$$N = \frac{T_p}{T_s}$$

$$\text{error rate} = 2/3\ \pi^3 \frac{\theta^2 T_s^2}{T_p^2} \times \frac{1}{T_p} \text{ radians/sec.}$$

In terms of θ degrees and error rate in degrees/hour,

$$\text{error rate} = 1290 \frac{\theta^2}{T_p^3} Ts^2 \text{ deg/hr.}$$

Measured data indicate pitch and roll amplitudes of 3 deg with a period of 1.5 seconds. Then

$$\text{error rate} = 3400\ T_s^2 \text{ deg/hr.}$$

A sampling interval of 0.1 s is clearly insufficient. At 0.01 s, the error is reduced to 0.34 deg per hour. The input motion is, of course, exceptionally severe since a constant 90 deg phase relationship between pitch and roll is assumed.

APPENDIX E.--PROGRAM LISTING FOR SECTION 3, BODY OF REPORT

The results of Section 3 were obtained with the following computer program, LNDNAV. The listing shows a particular choice of input parameters which, of course, are changed to accommodate the various vehicle scenarios. The notation AMP(3,2) refers to the Z-axis amplitude during the second interval. FREQ(1,4) is the period of the X-axis motion during the fourth interval. T1, T2, T3, and T4 give the time at the end of each interval.

In the listing shown, some of the constants were preselected even though the ability of the operator to insert these constants in response to computer prompting is preserved.

```
03/11/80    15:57:58    AF            SYSTEMS REAL-TIME MONITOR-7.0

S E L   E X T E N D E D   F O R T R A N   I V   (R E V - 0 / 7 8 S E P 0 8)

LNONAV

                    PROGRAM LNONAV
                    REAL*8 CREF,CCMP,PHI,DLAT,DHDG,COSHD,SINHD,PI,PX,PY,PZ,DELT,T
                    REAL*8 EDUT,FRN,CMPR,DELR,DELX,DELY,DELZ,DELXM,DELYM,DELZM,STEP
                    REAL*8 P1,A,B
                    DIMENSION CREF(3,3),CCMP (3,3),A(3),B(3),AMP(3,4),
                   1FREQ (3,4),F(3),ERN(3),DELR(3),CMPR(3),PHI(3,3)
              C
              C     SELECT CONSTANTS
              C
                    AMP(1,1)=0.0
                    AMP(1,2)=10.0
                    AMP(1,3)=0.0
                    AMP(1,4)=0.0
                    AMP(2,1)=10.0
                    AMP(2,2)=0.0
                    AMP(2,3)=10.0
                    AMP(2,4)=0.0
                    DO 500 J=1,4
                    FREQ(1,J)=10.0
                    FREQ(2,J)=11.0
                    AMP(3,J)=90.0
                500 FREQ(3,J)=1200.0
                    T1=100.0
                    T2=200.0
                    T3=300.0
                    T4=600.0
                    DLAT=45.
                    DHDG=0.
                  2 FORMAT (3F6.2)
                 13 FORMAT (D6.2)
                    WRITE (5,15)
                 15 FORMAT (' ENTER QUANTIZING INTERVAL IN TENTHS OF SECONDS '/)
                    READ (5,16) INT
                16     FORMAT(I3)
                    WRITE (5,17)
                 17 FORMAT (' ENTER NUMBER OF ITERATIONS OF SCENARIO '/)
                    READ (5,16) LIMIT
                    WRITE (5,18)
                 18 FORMAT (' ENTER PRINT INTERVAL IN SECONDS '/)
                    READ (5,16) M
                    WRITE (5,19)
                 19 FORMAT (' ENTER X,Y,Z SENSOR SCALE FACTORS '/)
                    READ (5,2) A(1), A(2), A(3)
                    WRITE (5,20)
                 20 FORMAT (' ENTER X,Y,Z SENSOR BIAS IN DEGREES PER HOUR'/)
                    READ (5,2) B(1), B(2), B(3)
                    WRITE (5,21)
                 21 FORMAT (' ENTER MINIMUM ANGLE STEP IN DEGREES '/)
                    READ(5,13)STEP
              C
              C     PRINT OUT CONDITIONS
              C
                    WRITE (7,30) (AMP (1,J),J=1,4)
                 30 FORMAT ('1X AMPLITUDES =  ' 4F9.5)
```

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```
      WRITE (7,31) (FREQ(1,J), J=1,4)
   31 FORMAT (' X PERIODS =  ' 4F9.5)
      WRITE (7,32)(AMP (2,J),J=1,4)
   32 FORMAT (' Y AMPLITUDES=  ' 4F9.5)
      WRITE (7,33)(FREQ (2,J),J=1,4)
   33   FORMAT (' Y PERIODS =  ' 4F9.5)
      WRITE (7,34)(AMP (3,J), J=1,4)
   34 FORMAT (' Z AMPLITUDES=  ' 4F9.5 )
      WRITE (7,35)(FREQ (3,J), J=1,4 )
   35 FORMAT(' Z PERIODS=  ' 4F9.2)
      WRITE (7,36) T1,T2,T3,T4
   36    FORMAT (' TIME AT END OF EACH INTERVAL IN SECONDS=  '4F9.0)
      WRITE (7,37) INT
   37 FORMAT (' QUANTIZING INTERVAL IN TENTHS OF SECONDS =  ' I4)
      WRITE (7,29) STEP
   29 FORMAT (' MINIMUM ANGLE STEP=  ' F9.5)
      WRITE (7,38) DHDG
   38    FORMAT (' INITIAL HEADING =  'F9.0)
      WRITE (7,39) DLAT
   39    FORMAT (' LATITUDE=  ' F9.0)
      WRITE (7,40) A(1), A(2), A(3)
   40 FORMAT (' SENSOR X,Y,Z SCALE FACTORS =  '3F9.3)
      WRITE (7,41) B(1), B(2), B(3)
   41 FORMAT (' SENSOR X,Y,Z BIAS IN DEGREES PER HOUR=  '3F9.3///)
      WRITE(7,42)
   42 FORMAT (5X,'TIME(MINUTES)',5X,'HEADING',5X,
     1'TILT', 5X,'ERROR',8X,'DETCREF',8X,'DETCOMP'///)
C
C     CALCULATE EARTHRATE OF NAV FRAME (ERN)
C
      PI=3.1415926536 D+0
      PI=PI
      DLAT=DLAT*PI/180.0 D+0
      EUDT=PI/43200.0 D+0
      ERN(1)=DCOS (DLAT)*EDUT
      ERN(2)= 0.0 D+0
      ERN(3)= -DSIN(DLAT)*EUDT
C
C     INITIALIZE DIRECTION COSINE MATRICES
C
      DHDG=DHDG*PI/180.0 D+0
      CUSHD=DCOS(DHDG)
      SINHD=DSIN(DHDG)
      CREF=0.0 D+0
      CREF(1,1)=CUSHD
      CREF(1,2)= -SINHD
      CREF(2,1)= SINHD
      CREF(2,2)=CUSHD
      CREF(3,3)=1.0 D+0
      CCMP=0.0 D+0
      CCMP(1,1)=CUSHD
      CCMP(1,2)= -SINHD
      CCMP(2,1)=SINHD
      CCMP(2,2)=CUSHD
```

```
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S E L  E X T E N D E D  F O R T R A N  I V  (R E V - 0 / / 8 S E P 0 8)

LNDWAV

                                CCMP(3,3)= 1.0 D+0
                          C
                          C     CONVERT INPUT AMPLITUDES TO RADIANS
                          C
                                DO 50 J=1,3
                                DO 50 K=1,4
                             50 AMP(J,K)=AMP(J,K)* PI/180.0
                                STEP=STEP*PI/180.0
                          C
                          C     CONVERT INPUT PERIODS TO RADIAN FREQUENCIES
                                DO 60 J=1,3
                                DO 60 K=1,4
                             60    FREQ  (J,K)=2*PI/FREQ(J,K)
                          C
                          C
                          C     INITIALIZE PRINT INTERAL COUNTER IN TENTHS OF SECONDS
                          C
                                N=10*N
                                NPRINT=N
                          C
                          C     BEGIN MAIN PROGRAM
                          C
                                PX=0.0D+0
                                PY=0.0D+0
                                PZ=0.0D+0
                                MAX=T4*10.0
                                M=0
                                DO 210 ITER=1,LIMIT
                                DO 200 NT=INT,MAX,INT
                                T=NT
                                T=0.1*T
                                IF(T.LT.T1) GO TO 110
                                IF (T.LT.T2) GO TO 120
                                IF (T.LT.T3) GO TO 130
                                DO 105 J=1,3
                            105 F(J)=AMP(J,4)*SIN(FREQ(J,4)*T)
                                GO TO 140
                            110 DO 111 J=1,3
                            111 F(J)=AMP(J,1)*SIN(FREQ(J,1)*T)
                                GO TO 140
                            120 DO 121 J=1,3
                            121 F(J)=AMP(J,2)*SIN(FREQ(J,2)*T)
                                GO TO 140
                            130 DO 131 J=1,3
                            131 F(J)=AMP(J,3)*SIN(FREQ (J,3)*T)
                            140 NPRINT=NPRINT -INT
                                M=M+1
                                DELT=M*INT/10.0D+0
                                DELX=F(1)-PX
                                DELY=F(2)-PY
                                DELZ=F(3)-PZ
                                IF(DABS(DELX).LT.STEP)DELX=0.0D+0
                                IF(DABS(DELY).LT.STEP)DELY=0.0D+0
                                IF(DABS(DELZ).LT.STEP)DELZ=0.0D+0
```

```
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S E L  E X T E N D E D  F O R T R A N  I V  (R E V - 0 / 7 8 S E P 0 6)

LNDNAV

                    IF (DELX + DELY + DELZ .EQ.0.  ) GO TO 160
                      PX=PX + DELX
                    PY=PY + DELY
                    PZ=PZ + DELZ
              C
              C     BEGIN MATRIX UPDATE ROUTINE
              C
              C     COMPUTE ANGLE INCREMENTS BETWEEN STEPS AS SEEN BY COMPUTER
              C
                    DO 151 J=1,3
                    CMPR(J)=B(J)*PI/648000.0D+0
                    DO 150 K=1,3
                    CMPR(J)=CMPR(J)+(A(J)*CREF(K,J)-CCMP(K,J))*ERN(K)
                150 CONTINUE
                    DELR(J)=CMPR(J)*DELT
                151 CONTINUE
              C
              C     UPDATE CCMP WITH PHI MATRIX
              C
                    CALL MAT(DELR(1),DELR(2),DELR(3),PHI)
                    CALL UPDATE (CCMP,PHI)
              C
              C     UPDATE CREF WITH ANGLE STEP
              C
                    CALL MAT(DELX,DELY,DELZ,PHI)
                    CALL UPDATE (CREF,PHI)
              C
              C     UPDATE CCMP WITH MEASURED ANGLE STEP
              C
                    DELXM=A(1)*DELX
                    DELYM=A(2)*DELY
                    DELZM=A(3)*DELZ
                    CALL MAT(DELXM,DELYM,DELZM,PHI)
                    CALL UPDATE(CCMP,PHI)
              C
              C     END OF MATRIX UPDATE ROUTINE
              C
                    M=0
                160 IF(NPRINT .GT. 0) GO TO 200
                    TIME= NT+(ITER -1)*MAX
                    TIMEM=TIME/600.0
                    SHDG=DATAN2(CREF(2,1),CREF(1,1))*180.0/PI
                    CREF33=CREF(3,3)
                    STILT=ACOS(CREF33)*180.0/PI
                    ERROR=DATAN2(CCMP(2,1),CCMP(1,1))*180.0/PI-SHDG
                    DETR=CREF(1,1)*(CREF(2,2)*CREF(3,3)-CREF(2,3)*
                   1CREF(3,2))-CREF(1,2)*(CREF(2,1)*CREF(3,3)
                   2-CREF(2,3)*CREF(3,1)) + CREF(1,3)*(CREF(2,1)*CREF(3,2)
                   3-CREF(2,2)*CREF(3,1))
                    DETC=CCMP(1,1)*(CCMP(2,2)*CCMP(3,3)-CCMP(2,3)*
                   1CCMP(3,2))-CCMP(1,2)*(CCMP(2,1)*CCMP(3,3)
                   2-CCMP(2,3)*CCMP(3,1)) + CCMP(1,3)*(CCMP(2,1)*CCMP(3,2)
                   3-CCMP(2,2)*CCMP(3,1))
                    WRITE (7,170) TIMEM,SHDG,STILT,ERROR,DETR,DETC
```

```
03/11/80    15:57:58    AF            SYSTEMS REAL-TIME MONITOR-7.0

S E L  E X T E N D E D  F O R T R A N  I V  (R E V - 0 / 7 8 S E P 0 6)

LNDNAV

                                  170 FORMAT (9X,F6.2,6X,3F10.2,2F15.8)
                                  400 FORMAT (3F15.8)
                                      NPRINT=N
                                  200 CONTINUE
                                  210 CONTINUE
                                      END
```

```
   03/11/80    15:57:58    AF            SYSTEMS REAL-TIME MONITOR-7.0

SEL EXTENDED FORTRAN IV (REV-0/78SEP08)

MAT

1                                       SUBROUTINE MAT (X,Y,Z,C)
2                                       IMPLICIT REAL*8(A-Z)
3                                       DIMENSION C(3,3)
4                                       XSQ=X*X
5                                       YSQ=Y*Y
6                                       ZSQ=Z*Z
7                                       BSQ=XSQ+YSQ+ZSQ
8                                       S=1.0D+0
9                                       Q=0.0D+0
10                                      IF (BSQ  .EQ. 0) GO TO 350
11                                      BETA=DSQRT(BSQ)
12                                      S=(DSIN(BETA))/BETA
13                                      Q=(1-DCOS(BETA))/BSQ
14                                  350 C(1,1)=1.0 D+0-Q*(YSQ+ZSQ)
15                                      C(1,2)=Q*X*Y-S*Z
16                                      C(1,3)=Q*X*Z+S*Y
17                                      C(2,1)=Q*X*Y+S*Z
18                                      C(2,2)=1.0 D+0-Q*(XSQ+ZSQ)
19                                      C(2,3)=Q*Y*Z-S*X
20                                      C(3,1)=Q*X*Z-S*Y
21                                      C(3,2)=Q*Y*Z + S*X
22                                      C(3,3)=1.0 D+0 - Q*(XSQ+YSQ)
23                                      RETURN
24                                      END
```

SEL EXTENDED FORTRAN IV (REV-0//05EP00)

UPDATE

```
1                       SUBROUTINE UPDATE (C,PHI)
2                       REAL *8 C,PHI,TEMP
3                       DIMENSION C(3,3),PHI(5,5),TEMP(5,5)
4                       DO 300 J=1,5
5                       DO 300 K=1,5
6                       TEMP(J,K)=0.0 D+0
7                       DO 300 L=1,5
8                  300  TEMP(J,K)=TEMP(J,K)+C(J,L)*PHI(L,K)
9                       DO 310 J=1,5
10                      DO 310 K=1,5
11                 310  C(J,K)=TEMP(J,K)
12                      RETURN
13                      END
```


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APPENDIX F.--PRINTOUTS FOR SECTION 3

Printouts for a number of cases discussed in Section 3 are shown. These correspond to cases 3.4.2e, 3.4.2g, 3.4.3, 3.4.4(d), 3.4.5(b), and 3.4.6 of that section.

X AMPLITUDES = 10.00000 10.00000 0.00000 0.00000
X PERIODS = 10.00000 10.00000 10.00000 10.00000
Y AMPLITUDES= 10.00000 0.00000 10.00000 0.00000
Y PERIODS = 11.00000 11.00000 11.00000 11.00000
Z AMPLITUDES= 90.00000 90.00000 90.00000 90.00000
Z PERIODS= 1200.00 1200.00 1200.00 1200.00
TIME AT END OF EACH INTERVAL IN SECONDS= 100. 200. 300. 600.
QUANTIZING INTERVAL IN TENTHS OF SECONDS = 10
MINIMUM ANGLE STEP= 1.00000
INITIAL HEADING = 0.
LATITUDE= 45.
SENSOR X,Y,Z SCALE FACTORS = 1.050 1.050 1.050
SENSOR X,Y,Z BIAS IN DEGREES PER HOUR= 0.000 0.000 0.000

TIME(MINUTES)	HEADING	TILT	ERROR	DETCREF	DETCOMP
1.00	44.13	2.79	3.02	0.99999994	0.99999994
2.00	53.79	0.16	2.60	0.99999994	0.99999994
3.00	73.00	0.19	3.62	0.99999994	0.99999994
4.00	85.73	9.85	4.21	0.99999994	0.99999994
5.00	89.86	0.10	4.01	0.99999994	0.99999994
6.00	86.73	0.10	4.24	0.99999994	0.99999994
7.00	73.53	0.10	3.57	0.99999994	0.99999994
8.00	53.07	0.10	2.54	0.99999994	0.99999994
9.00	27.98	0.10	1.28	0.99999994	0.99999994
10.00	0.17	0.10	-0.12	0.99999994	0.99999994
11.00	44.30	2.76	2.90	0.99999994	0.99999994
12.00	53.90	0.26	2.56	0.99999994	0.99999994
13.00	73.17	0.29	3.49	0.99999994	0.99999994
14.00	85.91	9.79	4.07	0.99999994	0.99999994
15.00	90.03	0.20	4.28	0.99999994	0.99999994
16.00	86.90	0.20	4.12	0.99999994	0.99999994
17.00	73.71	0.20	3.45	0.99999994	0.99999994
18.00	53.25	0.20	2.41	0.99999994	0.99999994
19.00	28.16	0.20	1.15	0.99999994	0.99999994
20.00	0.35	0.20	-0.25	0.99999994	0.99999994
21.00	44.47	2.76	2.76	0.99999994	0.99999994
22.00	54.14	0.36	2.43	0.99999994	0.99999994
23.00	73.34	0.39	3.36	0.99999994	0.99999994
24.00	86.10	9.74	3.93	0.99999994	0.99999994
25.00	90.20	0.31	4.15	0.99999994	0.99999994
26.00	87.07	0.31	3.99	0.99999994	0.99999994
27.00	73.98	0.31	3.32	0.99999994	0.99999994
28.00	53.42	0.31	2.29	0.99999994	0.99999994
29.00	28.33	0.31	1.02	0.99999994	0.99999994
30.00	0.52	0.31	-0.36	0.99999994	0.99999994
31.00	44.63	2.77	2.66	0.99999994	0.99999994
32.00	54.31	0.46	2.30	0.99999994	0.99999994
33.00	73.51	0.50	3.24	0.99999994	0.99999994
34.00	86.29	9.69	3.79	0.99999994	0.99999994
35.00	90.37	0.41	4.02	0.99999994	0.99999994
36.00	87.25	0.41	3.86	0.99999994	0.99999994
37.00	74.05	0.41	3.19	0.99999994	0.99999994
38.00	53.59	0.41	2.15	0.99999994	0.99999994
39.00	28.50	0.41	0.89	0.99999994	0.99999994
40.00	0.69	0.41	-0.51	0.99999994	0.99999994
41.00	44.80	2.77	2.53	0.99999994	0.99999994

42.00	54.18	0.56	2.17	0.999999994	0.999999994
43.00	73.69	0.60	3.10	0.999999994	0.999999994
44.00	86.43	0.63	3.64	0.999999994	0.999999994
45.00	90.55	0.51	3.89	0.999999994	0.999999994
46.00	87.42	0.51	3.75	0.999999994	0.999999994
47.00	79.22	0.51	3.06	0.999999994	0.999999994
48.00	53.76	0.51	2.02	0.999999994	0.999999994
49.00	28.68	0.51	0.76	0.999999994	0.999999994
50.00	0.86	0.51	-0.65	0.999999994	0.999999994
51.00	49.97	2.78	2.40	0.999999994	0.999999994
52.00	84.66	0.67	2.03	0.999999994	0.999999994
53.00	73.86	0.70	2.97	0.999999994	0.999999994
54.00	86.66	9.56	3.49	0.999999994	0.999999994
55.00	90.72	0.61	3.76	0.999999994	0.999999994
56.00	87.59	0.61	3.59	0.999999994	0.999999994
57.00	74.40	0.61	2.92	0.999999994	0.999999994
58.00	53.94	0.61	1.89	0.999999994	0.999999994
59.00	28.85	0.61	0.62	0.999999994	0.999999994
60.00	1.04	0.61	-0.78	0.999999994	0.999999994
61.00	45.14	2.79	2.27	0.999999994	0.999999994
62.00	54.93	0.77	1.90	0.999999994	0.999999994
63.00	74.05	0.80	2.83	0.999999994	0.999999994
64.00	86.85	9.53	3.35	0.999999994	0.999999994
65.00	90.89	0.72	3.62	0.999999994	0.999999994
66.00	87.76	0.72	3.45	0.999999994	0.999999994
67.00	74.57	0.72	2.78	0.999999994	0.999999994
68.00	54.11	0.72	1.75	0.999999994	0.999999994
69.00	29.02	0.72	0.66	0.999999994	0.999999994
70.00	1.21	0.72	-0.92	0.999999994	0.999999994
71.00	45.30	2.80	2.13	0.999999994	0.999999994
72.00	55.00	0.87	1.76	0.999999994	0.999999994
73.00	74.20	0.90	2.67	0.999999994	0.999999994
74.00	87.05	9.46	3.19	0.999999994	0.999999994
75.00	91.00	0.82	3.48	0.999999994	0.999999994
76.00	87.94	0.82	3.32	0.999999994	0.999999994
77.00	74.74	0.82	2.64	0.999999994	0.999999994
78.00	54.26	0.82	1.61	0.999999994	0.999999994
79.00	29.20	0.82	0.34	0.999999994	0.999999994
80.00	1.30	0.82	-1.06	0.999999994	0.999999994
81.00	45.47	2.82	2.00	0.999999994	0.999999994
82.00	55.17	0.97	1.62	0.999999994	0.999999994
83.00	74.37	1.01	2.55	0.999999994	0.999999994
84.00	87.22	9.43	3.04	0.999999994	0.999999994
85.00	91.25	0.92	3.34	0.999999994	0.999999994
86.00	88.11	0.92	3.17	0.999999994	0.999999994
87.00	74.91	0.92	2.50	0.999999994	0.999999994
88.00	54.45	0.92	1.66	0.999999994	0.999999994
89.00	29.37	0.92	0.20	0.999999994	0.999999994
90.00	1.50	0.92	-1.21	0.999999994	0.999999994
91.00	45.64	2.84	1.86	0.999999994	0.999999994
92.00	55.35	1.07	1.47	0.999999994	0.999999994
93.00	74.55	1.11	2.81	0.999999994	0.999999994
94.00	87.41	9.36	2.89	0.999999994	0.999999994
95.00	91.41	1.02	3.20	0.999999994	0.999999994
96.00	88.20	1.02	3.03	0.999999994	0.999999994
97.00	75.06	1.02	2.36	0.999999994	0.999999994
98.00	54.63	1.02	1.32	0.999999994	0.999999994
99.00	29.54	1.02	0.05	0.999999994	0.999999994
100.00	1.73	1.02	-1.35	0.999999994	0.999999994

101.00	45.91	2.87	1.72	0.99999994	0.99999994
102.00	55.52	1.18	1.35	0.99999994	0.99999994
103.00	74.72	1.21	2.21	0.99999994	0.99999994
104.00	87.59	9.35	2.75	0.99999994	0.99999994
105.00	91.56	1.12	3.05	0.99999994	0.99999994
106.00	88.45	1.12	2.89	0.99999994	0.99999994
107.00	75.26	1.12	2.21	0.99999994	0.99999994
108.00	54.80	1.12	1.17	0.99999994	0.99999994
109.00	29.71	1.12	-0.16	0.99999994	0.99999994
110.00	1.90	1.12	-1.50	0.99999994	0.99999994
111.00	45.96	2.90	1.57	0.99999994	0.99999994
112.00	55.69	1.28	1.18	0.99999994	0.99999994
113.00	74.89	1.31	2.12	0.99999994	0.99999994
114.00	87.78	9.28	2.56	0.99999994	0.99999994
115.00	91.75	1.25	2.90	0.99999994	0.99999994
116.00	88.62	1.25	2.74	0.99999994	0.99999994
117.00	75.43	1.25	2.06	0.99999994	0.99999994
118.00	54.97	1.25	1.05	0.99999994	0.99999994
119.00	29.84	1.25	-0.25	0.99999994	0.99999994
120.00	2.08	1.23	-1.65	0.99999994	0.99999994
121.00	46.14	2.93	1.45	0.99999994	0.99999994
122.00	55.86	1.36	1.03	0.99999994	0.99999994
123.00	75.06	1.41	1.97	0.99999994	0.99999994
124.00	87.97	9.25	2.42	0.99999994	0.99999994
125.00	91.92	1.33	2.76	0.99999994	0.99999994
126.00	88.79	1.33	2.59	0.99999994	0.99999994
127.00	75.60	1.33	1.92	0.99999994	0.99999994
128.00	55.14	1.33	0.86	0.99999994	0.99999994
129.00	30.06	1.33	-0.40	0.99999994	0.99999994
130.00	2.25	1.33	-1.80	0.99999994	0.99999994
131.00	46.31	2.96	1.28	0.99999994	0.99999994
132.00	56.04	1.48	0.86	0.99999994	0.99999994
133.00	75.25	1.52	1.82	0.99999994	0.99999994
134.00	88.15	9.18	2.26	0.99999994	0.99999994
135.00	92.09	1.45	2.61	0.99999994	0.99999994
136.00	89.97	1.45	2.44	0.99999994	0.99999994
137.00	75.77	1.43	1.76	0.99999994	0.99999994
138.00	55.32	1.43	0.72	0.99999994	0.99999994
139.00	30.24	1.43	-0.55	0.99999994	0.99999994
140.00	2.45	1.45	-1.95	0.99999994	0.99999994
141.00	46.48	3.00	1.13	0.99999994	0.99999994
142.00	56.21	1.58	0.73	0.99999994	0.99999994
143.00	75.41	1.62	1.67	0.99999994	0.99999994
144.00	88.34	9.13	2.10	0.99999994	0.99999994
145.00	92.26	1.55	2.45	0.99999994	0.99999994
146.00	89.14	1.53	2.29	0.99999994	0.99999994
147.00	75.94	1.55	1.61	0.99999994	0.99999994
148.00	55.49	1.55	0.57	0.99999994	0.99999994
149.00	30.41	1.55	-0.70	0.99999994	0.99999994
150.00	2.60	1.55	-2.11	0.99999994	0.99999994
151.00	46.65	3.05	0.98	0.99999994	0.99999994
152.00	56.38	1.69	0.58	0.99999994	0.99999994
153.00	75.58	1.72	1.51	0.99999994	0.99999994
154.00	88.52	9.09	1.94	0.99999994	0.99999994
155.00	92.44	1.65	2.30	0.99999994	0.99999994
156.00	89.31	1.65	2.13	0.99999994	0.99999994
157.00	76.12	1.65	1.46	0.99999994	0.99999994
158.00	55.66	1.65	0.42	0.99999994	0.99999994
159.00	30.58	1.65	-0.86	0.99999994	0.99999994

160.00	2.71	1.65	-2.27	0.9999994	0.9999994
161.00	46.92	3.09	0.85	0.9999994	0.9999994
162.00	26.55	1.74	0.42	0.9999994	0.9999994
163.00	15.75	1.82	1.36	0.9999994	0.9999994
164.00	88.71	9.04	1.70	0.9999994	0.9999994
165.00	92.61	1.74	2.15	0.9999994	0.9999994
166.00	89.48	1.74	1.98	0.9999994	0.9999994
167.00	76.29	1.74	1.30	0.9999994	0.9999994
168.00	55.83	1.74	0.20	0.9999994	0.9999994
169.00	30.76	1.74	-1.02	0.9999994	0.9999994
170.00	2.95	1.74	-2.42	0.9999994	0.9999994
171.00	46.98	3.14	0.67	0.9999994	0.9999994
172.00	26.73	1.89	0.26	0.9999994	0.9999994
173.00	15.92	1.92	1.20	0.9999994	0.9999994
174.00	88.89	9.00	1.62	0.9999994	0.9999994
175.00	92.76	1.84	1.99	0.9999994	0.9999994
176.00	89.65	1.84	1.82	0.9999994	0.9999994
177.00	76.46	1.84	1.15	0.9999994	0.9999994
178.00	56.01	1.84	0.10	0.9999994	0.9999994
179.00	30.93	1.84	-1.18	0.9999994	0.9999994
180.00	3.12	1.84	-2.58	0.9999994	0.9999994

X AMPLITUDES = 10.00000 10.00000 0.00000 0.00000
X PERIODS = 10.00000 10.00000 10.00000 10.00000
Y AMPLITUDES= 10.00000 0.00000 10.00000 0.00000
Y PERIODS = 11.00000 11.00000 11.00000 11.00000
Z AMPLITUDES= 90.00000 90.00000 90.00000 90.00000
Z PERIODS= 1200.00 1200.00 1200.00 1200.00
TIME AT END OF EACH INTERVAL IN SECONDS= 100. 200. 300. 600.
QUANTIZING INTERVAL IN TENTHS OF SECONDS = 10
MINIMUM ANGLE STEP= 1.00000
INITIAL HEADING = 0.
LATITUDE= 45.
SENSOR X,Y,Z SCALE FACTORS = 1.020 1.020 1.020
SENSOR X,Y,Z BIAS IN DEGREES PER HOUR= 1.000 1.000 1.000

TIME(MINUTES)	HEADING	TILT	ERROR	DETCREF	DETCCMP
1.00	44.13	2.74	1.22	0.99999994	0.99999994
2.00	53.74	0.16	1.11	0.99999994	0.99999994
3.00	73.00	0.19	1.50	0.99999994	0.99999994
4.00	85.73	9.85	1.74	0.99999994	0.99999994
5.00	89.90	0.10	1.85	0.99999994	0.99999994
6.00	86.73	0.10	1.79	0.99999994	0.99999994
7.00	73.53	0.10	1.54	0.99999994	0.99999994
8.00	53.07	0.10	1.15	0.99999994	0.99999994
9.00	27.98	0.10	0.66	0.99999994	0.99999994
10.00	0.17	0.10	0.11	0.99999994	0.99999994
11.00	44.30	2.76	1.34	0.99999994	0.99999994
12.00	53.96	0.26	1.22	0.99999994	0.99999994
13.00	73.17	0.29	1.61	0.99999994	0.99999994
14.00	85.91	9.79	1.81	0.99999994	0.99999994
15.00	90.05	0.20	1.96	0.99999994	0.99999994
16.00	86.90	0.20	1.90	0.99999994	0.99999994
17.00	73.71	0.20	1.65	0.99999994	0.99999994
18.00	53.25	0.20	1.26	0.99999994	0.99999994
19.00	28.16	0.20	0.77	0.99999994	0.99999994
20.00	0.35	0.20	0.22	0.99999994	0.99999994
21.00	44.47	2.76	1.45	0.99999994	0.99999994
22.00	54.14	0.36	1.32	0.99999994	0.99999994
23.00	73.34	0.39	1.71	0.99999994	0.99999994
24.00	86.10	9.74	1.87	0.99999994	0.99999994
25.00	90.20	0.31	2.06	0.99999994	0.99999994
26.00	87.07	0.31	2.01	0.99999994	0.99999994
27.00	73.88	0.31	1.70	0.99999994	0.99999994
28.00	53.42	0.31	1.30	0.99999994	0.99999994
29.00	28.33	0.31	0.87	0.99999994	0.99999994
30.00	0.52	0.31	0.32	0.99999994	0.99999994
31.00	44.63	2.77	1.56	0.99999994	0.99999994
32.00	54.31	0.46	1.42	0.99999994	0.99999994
33.00	73.51	0.50	1.81	0.99999994	0.99999994
34.00	86.24	9.69	1.92	0.99999994	0.99999994
35.00	90.37	0.41	2.16	0.99999994	0.99999994
36.00	87.25	0.41	2.10	0.99999994	0.99999994
37.00	74.05	0.41	1.85	0.99999994	0.99999994
38.00	53.59	0.41	1.45	0.99999994	0.99999994
39.00	28.50	0.41	0.96	0.99999994	0.99999994
40.00	0.69	0.41	0.41	0.99999994	0.99999994
41.00	44.80	2.77	1.65	0.99999994	0.99999994

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42.00	54.46	0.56	1.51	0.999[illegible]	0.999[illegible]
43.00	15.69	0.60	1.90	0.999[illegible]	0.999[illegible]
44.00	86.48	9.65	1.97	0.999[illegible]	0.999[illegible]
45.00	90.52	0.51	2.25	0.999[illegible]	0.999[illegible]
46.00	87.42	0.51	2.19	0.999[illegible]	0.999[illegible]
47.00	14.22	0.51	1.94	0.999[illegible]	0.999[illegible]
48.00	53.76	0.51	1.54	0.999[illegible]	0.999[illegible]
49.00	28.66	0.51	1.04	0.999[illegible]	0.999[illegible]
50.00	0.80	0.51	0.09	0.999[illegible]	0.999[illegible]
51.00	44.97	2.76	1.75	0.999[illegible]	0.999[illegible]
52.00	54.66	0.67	1.60	0.999[illegible]	0.999[illegible]
53.00	73.86	0.70	1.99	0.999[illegible]	0.999[illegible]
54.00	86.66	9.58	2.01	0.999[illegible]	0.999[illegible]
55.00	90.72	0.61	2.35	0.999[illegible]	0.999[illegible]
56.00	87.59	0.61	2.28	0.999[illegible]	0.999[illegible]
57.00	74.90	0.61	2.02	0.999[illegible]	0.999[illegible]
58.00	53.94	0.61	1.62	0.999[illegible]	0.999[illegible]
59.00	28.85	0.61	1.12	0.999[illegible]	0.999[illegible]
60.00	1.04	0.61	0.57	0.999[illegible]	0.999[illegible]
61.00	45.14	2.79	1.85	0.999[illegible]	0.999[illegible]
62.00	54.85	0.77	1.67	0.999[illegible]	0.999[illegible]
63.00	74.03	0.80	2.07	0.999[illegible]	0.999[illegible]
64.00	86.85	9.53	2.05	0.999[illegible]	0.999[illegible]
65.00	90.89	0.72	2.41	0.999[illegible]	0.999[illegible]
66.00	87.76	0.72	2.35	0.999[illegible]	0.999[illegible]
67.00	74.57	0.72	2.10	0.999[illegible]	0.999[illegible]
68.00	54.11	0.72	1.69	0.999[illegible]	0.999[illegible]
69.00	29.02	0.72	1.19	0.999[illegible]	0.999[illegible]
70.00	1.21	0.72	0.63	0.999[illegible]	0.999[illegible]
71.00	45.30	2.80	1.90	0.999[illegible]	0.999[illegible]
72.00	55.00	0.87	1.74	0.999[illegible]	0.999[illegible]
73.00	74.20	0.90	2.14	0.999[illegible]	0.999[illegible]
74.00	87.05	9.46	2.07	0.999[illegible]	0.999[illegible]
75.00	91.06	0.82	2.48	0.999[illegible]	0.999[illegible]
76.00	87.94	0.82	2.42	0.999[illegible]	0.999[illegible]
77.00	74.74	0.82	2.16	0.999[illegible]	0.999[illegible]
78.00	54.26	0.82	1.70	0.999[illegible]	0.999[illegible]
79.00	29.20	0.82	1.25	0.999[illegible]	0.999[illegible]
80.00	1.30	0.82	0.69	0.999[illegible]	0.999[illegible]
81.00	45.47	2.82	1.97	0.999[illegible]	0.999[illegible]
82.00	55.17	0.97	1.81	0.999[illegible]	0.999[illegible]
83.00	74.37	1.01	2.20	0.999[illegible]	0.999[illegible]
84.00	87.22	9.45	2.09	0.999[illegible]	0.999[illegible]
85.00	91.25	0.92	2.54	0.999[illegible]	0.999[illegible]
86.00	88.11	0.92	2.40	0.999[illegible]	0.999[illegible]
87.00	74.91	0.92	2.22	0.999[illegible]	0.999[illegible]
88.00	54.45	0.92	1.81	0.999[illegible]	0.999[illegible]
89.00	29.37	0.92	1.30	0.999[illegible]	0.999[illegible]
90.00	1.50	0.92	0.74	0.999[illegible]	0.999[illegible]
91.00	45.64	2.84	2.03	0.999[illegible]	0.999[illegible]
92.00	55.35	1.07	1.80	0.999[illegible]	0.999[illegible]
93.00	74.55	1.11	2.25	0.999[illegible]	0.999[illegible]
94.00	87.41	9.50	2.11	0.999[illegible]	0.999[illegible]
95.00	91.41	1.02	2.59	0.999[illegible]	0.999[illegible]
96.00	88.28	1.02	2.54	0.999[illegible]	0.999[illegible]
97.00	75.04	1.02	2.28	0.999[illegible]	0.999[illegible]
98.00	54.63	1.02	1.80	0.999[illegible]	0.999[illegible]
99.00	29.54	1.02	1.35	0.999[illegible]	0.999[illegible]
100.00	1.73	1.02	0.74	0.999[illegible]	0.999[illegible]

101.00	45.91	2.47	2.03	0.999999994	0.999999994
102.00	55.52	1.10	1.91	0.999999994	0.999999994
103.00	78.74	1.21	2.37	0.999999994	0.999999994
104.00	87.59	9.33	2.11	0.999999994	0.999999994
105.00	91.58	1.12	2.64	0.999999994	0.999999994
106.00	88.45	1.12	2.58	0.999999994	0.999999994
107.00	75.26	1.12	2.32	0.999999994	0.999999994
108.00	54.80	1.12	1.91	0.999999994	0.999999994
109.00	29.71	1.14	1.39	0.999999994	0.999999994
110.00	1.90	1.12	0.82	0.999999994	0.999999994
111.00	45.96	2.90	2.13	0.999999994	0.999999994
112.00	55.69	1.28	1.95	0.999999994	0.999999994
113.00	78.89	1.31	2.35	0.999999994	0.999999994
114.00	87.78	9.28	2.11	0.999999994	0.999999994
115.00	91.75	1.23	2.68	0.999999994	0.999999994
116.00	88.62	1.23	2.63	0.999999994	0.999999994
117.00	75.43	1.25	2.36	0.999999994	0.999999994
118.00	54.97	1.23	1.94	0.999999994	0.999999994
119.00	29.89	1.25	1.42	0.999999994	0.999999994
120.00	2.08	1.23	0.85	0.999999994	0.999999994
121.00	46.14	2.95	2.17	0.999999994	0.999999994
122.00	55.90	1.38	1.98	0.999999994	0.999999994
123.00	78.09	1.41	2.38	0.999999994	0.999999994
124.00	87.97	9.25	2.11	0.999999994	0.999999994
125.00	91.92	1.33	2.72	0.999999994	0.999999994
126.00	88.79	1.33	2.66	0.999999994	0.999999994
127.00	75.60	1.33	2.40	0.999999994	0.999999994
128.00	55.14	1.35	1.97	0.999999994	0.999999994
129.00	30.00	1.35	1.44	0.999999994	0.999999994
130.00	2.25	1.33	0.86	0.999999994	0.999999994
131.00	46.31	2.90	2.20	0.999999994	0.999999994
132.00	56.04	1.48	2.01	0.999999994	0.999999994
133.00	78.23	1.52	2.41	0.999999994	0.999999994
134.00	88.15	9.18	2.10	0.999999994	0.999999994
135.00	92.09	1.43	2.75	0.999999994	0.999999994
136.00	88.97	1.43	2.69	0.999999994	0.999999994
137.00	75.77	1.43	2.42	0.999999994	0.999999994
138.00	55.52	1.45	1.99	0.999999994	0.999999994
139.00	30.24	1.45	1.45	0.999999994	0.999999994
140.00	2.43	1.43	0.89	0.999999994	0.999999994
141.00	46.48	3.00	2.22	0.999999994	0.999999994
142.00	56.21	1.58	2.05	0.999999994	0.999999994
143.00	75.41	1.62	2.43	0.999999994	0.999999994
144.00	89.34	9.15	2.06	0.999999994	0.999999994
145.00	92.26	1.53	2.77	0.999999994	0.999999994
146.00	89.14	1.55	2.71	0.999999994	0.999999994
147.00	75.94	1.55	2.44	0.999999994	0.999999994
148.00	55.19	1.55	2.01	0.999999994	0.999999994
149.00	30.41	1.55	1.46	0.999999994	0.999999994
150.00	2.60	1.53	0.88	0.999999994	0.999999994
151.00	46.65	3.05	2.25	0.999999994	0.999999994
152.00	56.38	1.69	2.04	0.999999994	0.999999994
153.00	75.50	1.72	2.45	0.999999994	0.999999994
154.00	88.52	9.09	2.05	0.999999994	0.999999994
155.00	92.44	1.63	2.70	0.999999994	0.999999994
156.00	89.31	1.65	2.72	0.999999994	0.999999994
157.00	76.12	1.63	2.46	0.999999994	0.999999994
158.00	55.66	1.63	2.02	0.999999994	0.999999994
159.00	30.58	1.65	1.46	0.999999994	0.999999994

160.00	2.71	1.63	0.81	0.9999999994	0.9999999994
161.00	46.42	3.09	2.24	0.9999999994	0.9999999994
162.00	56.55	1.79	2.04	0.9999999994	0.9999999994
163.00	75.75	1.82	2.46	0.9999999994	0.9999999994
164.00	68.71	9.04	2.05	0.9999999994	0.9999999994
165.00	92.61	1.74	2.79	0.9999999994	0.9999999994
166.00	89.48	1.74	2.73	0.9999999994	0.9999999994
167.00	76.29	1.74	2.46	0.9999999994	0.9999999994
168.00	55.85	1.74	2.02	0.9999999994	0.9999999994
169.00	30.76	1.74	1.45	0.9999999994	0.9999999994
170.00	2.95	1.74	0.80	0.9999999994	0.9999999994
171.00	46.98	3.14	2.24	0.9999999994	0.9999999994
172.00	56.75	1.89	2.04	0.9999999994	0.9999999994
173.00	75.92	1.92	2.46	0.9999999994	0.9999999994
174.00	68.89	9.00	1.99	0.9999999994	0.9999999994
175.00	92.78	1.84	2.80	0.9999999994	0.9999999994
176.00	89.65	1.84	2.74	0.9999999994	0.9999999994
177.00	76.46	1.84	2.46	0.9999999994	0.9999999994
178.00	56.01	1.84	2.01	0.9999999994	0.9999999994
179.00	30.95	1.84	1.44	0.9999999994	0.9999999994
180.00	3.12	1.84	0.84	0.9999999994	0.9999999994

```
X AMPLITUDES =    0.000000 10.000000  0.000000  0.000000
X PERIODS =   10.000000 10.000000 10.000000 10.000000
Y AMPLITUDES=   10.000000  0.000000 10.000000  0.000000
Y PERIODS =   11.000000 11.000000 11.000000 11.000000
Z AMPLITUDES=   90.000000 90.000000 90.000000 90.000000
Z PERIODS=    1200.00  1200.00  1200.00  1200.00
 TIME AT END OF EACH INTERVAL IN SECONDS=       100.     200.     300.     600.
QUANTIZING INTERVAL IN TENTHS OF SECONDS =          10
MINIMUM ANGLE STEP=     1.000000
 INITIAL HEADING =          0.
 LATITUDE=        45.
SENSOR X,Y,Z SCALE FACTORS =      1.020     1.020     1.020
SENSOR X,Y,Z BIAS IN DEGREES PER HOUR=      1.000     1.000     1.000
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TIME(MINUTES)	HEADING	TILT	ERROR	DETCREF	DETCOMP
1.00	27.62	2.78	0.56	0.99999994	0.99999994
2.00	52.52	0.16	1.06	0.99999994	0.99999994
3.00	71.73	0.19	1.45	0.99999994	0.99999994
4.00	84.46	9.86	1.69	0.99999994	0.99999994
5.00	88.59	0.09	1.80	0.99999994	0.99999994
6.00	85.46	0.09	1.75	0.99999994	0.99999994
7.00	72.27	0.09	1.50	0.99999994	0.99999994
8.00	51.81	0.09	1.10	0.99999994	0.99999994
9.00	26.72	0.09	0.61	0.99999994	0.99999994
10.00	-1.09	0.09	0.07	0.99999994	0.99999994
11.00	25.52	2.72	0.63	0.99999994	0.99999994
12.00	51.43	0.25	1.13	0.99999994	0.99999994
13.00	70.65	0.28	1.52	0.99999994	0.99999994
14.00	83.38	9.85	1.72	0.99999994	0.99999994
15.00	87.49	0.19	1.87	0.99999994	0.99999994
16.00	84.37	0.19	1.82	0.99999994	0.99999994
17.00	71.17	0.19	1.56	0.99999994	0.99999994
18.00	50.71	0.19	1.17	0.99999994	0.99999994
19.00	25.62	0.19	0.68	0.99999994	0.99999994
20.00	-2.19	0.19	0.13	0.99999994	0.99999994
21.00	25.42	2.66	0.70	0.99999994	0.99999994
22.00	50.35	0.34	1.19	0.99999994	0.99999994
23.00	69.54	0.37	1.56	0.99999994	0.99999994
24.00	82.30	9.83	1.74	0.99999994	0.99999994
25.00	86.40	0.28	1.93	0.99999994	0.99999994
26.00	83.27	0.28	1.87	0.99999994	0.99999994
27.00	70.08	0.28	1.62	0.99999994	0.99999994
28.00	49.62	0.28	1.22	0.99999994	0.99999994
29.00	24.53	0.28	0.73	0.99999994	0.99999994
30.00	-3.28	0.28	0.19	0.99999994	0.99999994
31.00	24.32	2.60	0.76	0.99999994	0.99999994
32.00	49.24	0.43	1.24	0.99999994	0.99999994
33.00	68.44	0.47	1.65	0.99999994	0.99999994
34.00	81.22	9.81	1.75	0.99999994	0.99999994
35.00	85.30	0.37	1.98	0.99999994	0.99999994
36.00	82.16	0.37	1.93	0.99999994	0.99999994
37.00	68.98	0.37	1.67	0.99999994	0.99999994
38.00	48.52	0.37	1.27	0.99999994	0.99999994
39.00	23.43	0.37	0.78	0.99999994	0.99999994
40.00	-4.38	0.37	0.25	0.99999994	0.99999994
41.00	23.23	2.54	0.81	0.99999994	0.99999994

42.00	48.14	0.53	1.29	0.999999944	0.999999944
43.00	67.35	0.50	1.60	0.999999944	0.999999944
44.00	80.14	0.79	1.70	0.999999944	0.999999944
45.00	84.21	0.47	2.05	0.999999944	0.999999944
46.00	81.06	0.47	1.91	0.999999944	0.999999944
47.00	67.89	0.47	1.72	0.999999944	0.999999944
48.00	47.45	0.47	1.32	0.999999944	0.999999944
49.00	22.34	0.47	0.82	0.999999944	0.999999944
50.00	-5.47	0.47	0.27	0.999999944	0.999999944
51.00	22.13	2.48	0.85	0.999999944	0.999999944
52.00	47.05	0.62	1.33	0.999999944	0.999999944
53.00	66.25	0.65	1.72	0.999999944	0.999999944
54.00	79.06	9.71	1.76	0.999999944	0.999999944
55.00	83.11	0.50	2.00	0.999999944	0.999999944
56.00	79.94	0.56	2.01	0.999999944	0.999999944
57.00	66.79	0.56	1.75	0.999999944	0.999999944
58.00	46.33	0.56	1.33	0.999999944	0.999999944
59.00	21.24	0.56	0.86	0.999999944	0.999999944
60.00	-6.57	0.50	0.30	0.999999944	0.999999944
61.00	21.03	2.42	0.88	0.999999944	0.999999944
62.00	45.95	0.71	1.36	0.999999944	0.999999944
63.00	65.10	0.74	1.75	0.999999944	0.999999944
64.00	77.90	9.70	1.75	0.999999944	0.999999944
65.00	82.02	0.65	2.09	0.999999944	0.999999944
66.00	78.99	0.65	2.04	0.999999944	0.999999944
67.00	65.70	0.65	1.76	0.999999944	0.999999944
68.00	45.24	0.65	1.38	0.999999944	0.999999944
69.00	20.12	0.65	0.86	0.999999944	0.999999944
70.00	-7.66	0.65	0.35	0.999999944	0.999999944
71.00	19.95	2.36	0.90	0.999999938	0.999999938
72.00	44.86	0.81	1.39	0.999999944	0.999999944
73.00	64.00	0.84	1.78	0.999999944	0.999999944
74.00	76.90	9.75	1.73	0.999999944	0.999999944
75.00	80.92	0.75	2.12	0.999999944	0.999999944
76.00	77.79	0.75	2.00	0.999999944	0.999999944
77.00	64.60	0.75	1.81	0.999999944	0.999999944
78.00	44.14	0.75	1.40	0.999999944	0.999999944
79.00	19.05	0.75	0.90	0.999999944	0.999999944
80.00	-9.76	0.75	0.34	0.999999944	0.999999944
81.00	18.83	2.30	0.92	0.999999944	0.999999944
82.00	43.71	0.90	1.41	0.999999944	0.999999944
83.00	62.91	0.93	1.80	0.999999944	0.999999944
84.00	75.82	9.74	1.72	0.999999944	0.999999944
85.00	79.83	0.84	2.14	0.999999944	0.999999944
86.00	76.70	0.84	2.00	0.999999944	0.999999944
87.00	63.51	0.84	1.82	0.999999944	0.999999944
88.00	43.05	0.84	1.41	0.999999944	0.999999944
89.00	17.90	0.84	0.91	0.999999944	0.999999944
90.00	-9.95	0.84	0.35	0.999999944	0.999999944
91.00	17.74	2.24	0.93	0.999999944	0.999999944
92.00	42.67	0.99	1.42	0.999999944	0.999999944
93.00	61.87	1.02	1.81	0.999999944	0.999999944
94.00	74.74	9.74	1.69	0.999999944	0.999999944
95.00	78.73	0.95	2.15	0.999999944	0.999999944
96.00	75.60	0.95	2.09	0.999999944	0.999999944
97.00	62.41	0.95	1.83	0.999999944	0.999999944
98.00	41.95	0.95	1.42	0.999999944	0.999999944
99.00	16.86	0.95	0.91	0.999999944	0.999999944
100.00	-10.95	0.95	0.35	0.999999944	0.999999944

101.00	15.64	2.19	0.93	0.999999994	0.999999994	0.999999994
102.00	41.50	1.09	1.42	0.999999994	0.999999994	0.999999994
103.00	60.76	1.12	1.41	0.999999994	0.999999994	0.999999994
104.00	73.67	9.74	1.60	0.999999994	0.999999994	0.999999994
105.00	77.64	1.03	2.15	0.999999994	0.999999994	0.999999994
106.00	74.51	1.03	2.09	0.999999994	0.999999994	0.999999994
107.00	61.32	1.03	1.83	0.999999994	0.999999994	0.999999994
108.00	40.86	1.03	1.42	0.999999994	0.999999994	0.999999994
109.00	15.71	1.03	0.90	0.999999994	0.999999994	0.999999994
110.00	-12.04	1.03	0.34	0.999999994	0.999999994	0.999999994
111.00	15.54	2.15	0.92	0.999999994	0.999999994	0.999999994
112.00	40.48	1.10	1.41	0.999999994	0.999999994	0.999999994
113.00	59.68	1.21	1.81	0.999999994	0.999999994	0.999999994
114.00	72.59	9.74	1.62	0.999999994	0.999999994	0.999999994
115.00	76.54	1.12	2.15	0.999999994	0.999999994	0.999999994
116.00	73.41	1.12	2.09	0.999999994	0.999999994	0.999999994
117.00	60.22	1.12	1.82	0.999999994	0.999999994	0.999999994
118.00	39.76	1.12	1.41	0.999999994	0.999999994	0.999999994
119.00	14.68	1.12	0.89	0.999999994	0.999999994	0.999999994
120.00	-13.14	1.12	0.33	0.999999994	0.999999994	0.999999994
121.00	14.44	2.06	0.90	0.999999994	0.999999994	0.999999994
122.00	39.39	1.27	1.40	0.999999994	0.999999994	0.999999994
123.00	58.59	1.30	1.80	0.999999994	0.999999994	0.999999994
124.00	71.51	9.74	1.58	0.999999994	0.999999994	0.999999994
125.00	75.44	1.21	2.13	0.999999994	0.999999994	0.999999994
126.00	72.32	1.21	2.06	0.999999994	0.999999994	0.999999994
127.00	59.15	1.21	1.81	0.999999994	0.999999994	0.999999994
128.00	38.67	1.21	1.39	0.999999994	0.999999994	0.999999994
129.00	13.58	1.21	0.87	0.999999994	0.999999994	0.999999994
130.00	-14.23	1.21	0.31	0.999999994	0.999999994	0.999999994
131.00	13.35	2.05	0.87	0.999999994	0.999999994	0.999999994
132.00	38.29	1.37	1.78	0.999999994	0.999999994	0.999999994
133.00	57.49	1.40	1.78	0.999999994	0.999999994	0.999999994
134.00	70.45	9.75	1.53	0.999999994	0.999999994	0.999999994
135.00	74.35	1.30	2.12	0.999999994	0.999999994	0.999999994
136.00	71.22	1.30	2.06	0.999999994	0.999999994	0.999999994
137.00	58.03	1.30	1.79	0.999999994	0.999999994	0.999999994
138.00	37.51	1.30	1.36	0.999999994	0.999999994	0.999999994
139.00	12.49	1.30	0.84	0.999999994	0.999999994	0.999999994
140.00	-15.35	1.30	0.27	0.999999994	0.999999994	0.999999994
141.00	12.25	1.96	0.84	0.999999994	0.999999994	0.999999994
142.00	37.20	1.46	1.35	0.999999994	0.999999994	0.999999994
143.00	56.40	1.49	1.75	0.999999994	0.999999994	0.999999994
144.00	69.35	9.75	1.48	0.999999994	0.999999994	0.999999994
145.00	73.25	1.40	2.09	0.999999994	0.999999994	0.999999994
146.00	70.13	1.40	2.03	0.999999994	0.999999994	0.999999994
147.00	56.95	1.40	1.78	0.999999994	0.999999994	0.999999994
148.00	36.48	1.40	1.33	0.999999994	0.999999994	0.999999994
149.00	11.39	1.40	0.80	0.999999994	0.999999994	0.999999994
150.00	-16.42	1.40	0.24	0.999999994	0.999999994	0.999999994
151.00	11.15	1.93	0.79	0.999999994	0.999999994	0.999999994
152.00	36.10	1.55	1.31	0.999999994	0.999999994	0.999999994
153.00	55.30	1.56	1.76	0.999999994	0.999999994	0.999999994
154.00	68.27	9.76	1.42	0.999999994	0.999999994	0.999999994
155.00	72.10	1.49	2.06	0.999999994	0.999999994	0.999999994
156.00	69.05	1.49	2.00	0.999999994	0.999999994	0.999999994
157.00	55.94	1.49	1.73	0.999999994	0.999999994	0.999999994
158.00	35.39	1.49	1.29	0.999999994	0.999999994	0.999999994
159.00	10.30	1.49	0.75	0.999999994	0.999999994	0.999999994

160.00	-17.52	1.49	0.19	0.999999994	0.999999994
161.00	10.06	1.86	0.74	0.999999994	0.999999994
162.00	35.01	1.64	1.27	0.999999994	0.999999994
163.00	58.21	1.67	1.68	0.999999994	0.999999994
164.00	67.19	9.76	1.35	0.999999994	0.999999994
165.00	71.06	1.56	2.02	0.999999994	0.999999994
166.00	67.94	1.56	1.96	0.999999994	0.999999994
167.00	54.74	1.56	1.68	0.999999994	0.999999994
168.00	34.29	1.58	1.24	0.999999994	0.999999994
169.00	9.21	1.56	0.70	0.999999994	0.999999994
170.00	-18.61	1.56	0.14	0.999999994	0.999999994
171.00	8.96	1.84	0.68	0.999999994	0.999999994
172.00	33.92	1.74	1.22	0.999999994	0.999999994
173.00	53.11	1.76	1.63	0.999999994	0.999999994
174.00	66.11	9.80	1.28	0.999999994	0.999999994
175.00	69.97	1.67	1.98	0.999999994	0.999999994
176.00	66.84	1.67	1.91	0.999999994	0.999999994
177.00	53.65	1.67	1.63	0.999999994	0.999999994
178.00	33.20	1.67	1.18	0.999999994	0.999999994
179.00	8.11	1.67	0.64	0.999999994	0.999999994
180.00	-19.71	1.67	0.00	0.999999994	0.999999994

X AMPLITUDES = 10.00000 10.00000 10.00000 10.00000
X PERIODS = 2.50000 2.50000 2.50000 2.50000
Y AMPLITUDES= 5.00000 5.00000 5.00000 5.00000
Y PERIODS = 2.50000 2.50000 2.50000 2.50000
Z AMPLITUDES= 90.00000 90.00000 0.00000 0.00000
Z PERIODS= 400.00 400.00 400.00 400.00
TIME AT END OF EACH INTERVAL IN SECONDS= 100. 200. 300. 600.
QUANTIZING INTERVAL IN TENTHS OF SECONDS = 3
MINIMUM ANGLE STEP= 1.00000
INITIAL HEADING = 0.
LATITUDE= 45.
SENSOR X,Y,Z SCALE FACTORS = 1.020 1.020 1.020
SENSOR X,Y,Z BIAS IN DEGREES PER HOUR= 1.000 1.000 1.000

TIME(MINUTES)	HEADING	TILT	ERROR	DETCREF	DETCOMP
1.00	72.39	1.71	1.44	0.99999994	0.99999994
2.00	65.27	1.70	1.71	0.99999994	0.99999994
3.00	28.24	0.45	0.61	0.99999994	0.99999994
4.00	0.90	0.21	0.09	0.99999994	0.99999994
5.00	1.12	0.23	0.11	0.99999994	0.99999994
6.00	1.35	0.27	0.13	0.99999994	0.99999994
7.00	1.57	0.32	0.16	0.99999994	0.99999994
8.00	1.80	0.37	0.18	0.99999994	0.99999994
9.00	2.02	0.43	0.20	0.99999994	0.99999994
10.00	2.25	0.49	0.22	0.99999994	0.99999994
11.00	74.64	1.29	1.66	0.99999994	0.99999994
12.00	67.52	1.21	1.93	0.99999994	0.99999994
13.00	30.48	0.15	0.83	0.99999994	0.99999994
14.00	3.15	0.64	0.31	0.99999994	0.99999994
15.00	3.37	0.69	0.33	0.99999994	0.99999994
16.00	3.60	0.75	0.35	0.99999994	0.99999994
17.00	3.82	0.80	0.37	0.99999994	0.99999994
18.00	4.05	0.86	0.40	0.99999994	0.99999994
19.00	4.27	0.92	0.42	0.99999994	0.99999994
20.00	4.50	0.98	0.44	0.99999994	0.99999994
21.00	76.89	0.93	1.87	0.99999994	0.99999994
22.00	89.77	0.74	2.14	0.99999994	0.99999994
23.00	52.73	0.56	1.04	0.99999994	0.99999994
24.00	5.40	1.12	0.52	0.99999994	0.99999994
25.00	5.62	1.18	0.54	0.99999994	0.99999994
26.00	5.85	1.24	0.57	0.99999994	0.99999994
27.00	6.07	1.29	0.59	0.99999994	0.99999994
28.00	6.30	1.35	0.61	0.99999994	0.99999994
29.00	6.52	1.41	0.63	0.99999994	0.99999994
30.00	6.74	1.47	0.65	0.99999994	0.99999994
31.00	79.14	0.71	2.08	0.99999994	0.99999994
32.00	92.02	0.30	2.34	0.99999994	0.99999994
33.00	38.98	1.04	1.25	0.99999994	0.99999994
34.00	7.65	1.61	0.73	0.99999994	0.99999994
35.00	7.87	1.67	0.75	0.99999994	0.99999994
36.00	8.10	1.73	0.77	0.99999994	0.99999994
37.00	8.32	1.79	0.79	0.99999994	0.99999994
38.00	8.54	1.84	0.81	0.99999994	0.99999994
39.00	8.77	1.90	0.83	0.99999994	0.99999994
40.00	8.99	1.96	0.85	0.99999994	0.99999994
41.00	81.39	0.76	2.20	0.99999994	0.99999994

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42.00	94.27	0.34	2.54	0.999999994	0.999999994
43.00	37.24	1.55	1.46	0.999999994	0.999999994
44.00	9.90	2.11	0.95	0.999999994	0.999999994
45.00	10.12	2.16	0.95	0.999999994	0.999999994
46.00	10.34	2.22	0.97	0.999999994	0.999999994
47.00	10.57	2.28	0.99	0.999999994	0.999999994
48.00	10.79	2.33	1.01	0.999999994	0.999999994
49.00	11.02	2.39	1.03	0.999999994	0.999999994
50.00	11.24	2.45	1.05	0.999999994	0.999999994
51.00	83.65	1.00	2.47	0.999999994	0.999999994
52.00	96.52	0.76	2.72	0.999999994	0.999999994
53.00	59.49	2.02	1.66	0.999999994	0.999999994
54.00	12.15	2.60	1.13	0.999999994	0.999999994
55.00	12.37	2.65	1.15	0.999999994	0.999999994
56.00	12.60	2.71	1.17	0.999999994	0.999999994
57.00	12.82	2.77	1.19	0.999999994	0.999999994
58.00	13.05	2.82	1.21	0.999999994	0.999999994
59.00	13.27	2.88	1.23	0.999999994	0.999999994
60.00	13.50	2.94	1.25	0.999999994	0.999999994
61.00	85.90	1.48	2.65	0.999999994	0.999999994
62.00	98.77	1.26	2.89	0.999999994	0.999999994
63.00	41.75	2.50	1.80	0.999999994	0.999999994
64.00	14.40	3.09	1.32	0.999999994	0.999999994
65.00	14.62	3.14	1.34	0.999999994	0.999999994
66.00	14.85	3.20	1.36	0.999999994	0.999999994
67.00	15.07	3.26	1.38	0.999999994	0.999999994
68.00	15.30	3.31	1.40	0.999999994	0.999999994
69.00	15.52	3.37	1.42	0.999999994	0.999999994
70.00	15.75	3.43	1.44	0.999999994	0.999999994
71.00	88.10	1.91	2.82	0.999999994	0.999999994
72.00	101.02	1.74	3.05	0.999999994	0.999999994
73.00	44.01	2.99	2.05	0.999999994	0.999999994
74.00	16.65	3.58	1.51	0.999999994	0.999999994
75.00	16.88	3.65	1.53	0.999999994	0.999999994
76.00	17.10	3.69	1.55	0.999999994	0.999999994
77.00	17.33	3.75	1.58	0.999999994	0.999999994
78.00	17.55	3.80	1.58	0.999999994	0.999999994
79.00	17.78	3.86	1.60	0.999999994	0.999999994
80.00	18.00	3.91	1.62	0.999999994	0.999999994
81.00	90.42	2.36	2.98	0.999999994	0.999999994
82.00	103.27	2.22	3.20	0.999999994	0.999999994
83.00	46.28	3.47	2.23	0.999999994	0.999999994
84.00	18.91	4.07	1.69	0.999999994	0.999999994
85.00	19.13	4.12	1.71	0.999999994	0.999999994
86.00	19.36	4.18	1.72	0.999999994	0.999999994
87.00	19.58	4.23	1.74	0.999999994	0.999999994
88.00	19.81	4.29	1.76	0.999999994	0.999999994
89.00	20.04	4.34	1.78	0.999999994	0.999999994
90.00	20.26	4.40	1.80	0.999999994	0.999999994
91.00	92.67	2.82	3.13	0.999999994	0.999999994
92.00	105.52	2.71	3.34	0.999999994	0.999999994
93.00	48.54	3.90	2.41	0.999999994	0.999999994
94.00	21.17	4.50	1.86	0.999999994	0.999999994
95.00	21.39	4.61	1.88	0.999999994	0.999999994
96.00	21.62	4.67	1.90	0.999999994	0.999999994
97.00	21.84	4.72	1.92	0.999999994	0.999999994
98.00	22.07	4.77	1.93	0.999999994	0.999999994
99.00	22.30	4.83	1.95	0.999999994	0.999999994
100.00	22.52	4.88	1.97	0.999999994	0.999999994

101.00	99.95	3.20	3.27	0.999999994	0.999999994
102.00	107.76	3.19	3.46	0.999999994	0.999999994
103.00	50.81	4.44	2.50	0.999999994	0.999999994
104.00	25.45	5.04	2.05	0.999999994	0.999999994
105.00	24.65	5.09	2.05	0.999999994	0.999999994
106.00	23.98	5.15	2.07	0.999999994	0.999999994
107.00	24.10	5.20	2.08	0.999999994	0.999999994
108.00	24.43	5.25	2.10	0.999999994	0.999999994
109.00	24.56	5.31	2.12	0.999999994	0.999999994
110.00	24.78	5.36	2.13	0.999999994	0.999999994
111.00	97.19	3.74	3.40	0.999999994	0.999999994
112.00	110.01	3.67	3.56	0.999999994	0.999999994
113.00	53.08	4.92	2.75	0.999999994	0.999999994
114.00	25.69	5.52	2.20	0.999999994	0.999999994
115.00	25.92	5.58	2.21	0.999999994	0.999999994
116.00	26.14	5.63	2.23	0.999999994	0.999999994
117.00	26.37	5.68	2.25	0.999999994	0.999999994
118.00	26.60	5.74	2.26	0.999999994	0.999999994
119.00	26.82	5.79	2.28	0.999999994	0.999999994
120.00	27.05	5.84	2.29	0.999999994	0.999999994
121.00	99.45	4.21	3.51	0.999999994	0.999999994
122.00	112.26	4.15	3.67	0.999999994	0.999999994
123.00	55.36	5.40	2.91	0.999999994	0.999999994
124.00	27.90	6.01	2.36	0.999999994	0.999999994
125.00	28.18	6.06	2.37	0.999999994	0.999999994
126.00	28.41	6.11	2.39	0.999999994	0.999999994
127.00	28.64	6.16	2.40	0.999999994	0.999999994
128.00	28.87	6.21	2.42	0.999999994	0.999999994
129.00	29.09	6.27	2.44	0.999999994	0.999999994
130.00	29.32	6.32	2.45	0.999999994	0.999999994
131.00	101.70	4.67	3.61	0.999999994	0.999999994
132.00	114.50	4.65	3.76	0.999999994	0.999999994
133.00	57.64	5.87	3.07	0.999999994	0.999999994
134.00	30.23	6.49	2.51	0.999999994	0.999999994
135.00	30.45	6.54	2.53	0.999999994	0.999999994
136.00	30.68	6.59	2.54	0.999999994	0.999999994
137.00	30.91	6.64	2.56	0.999999994	0.999999994
138.00	31.14	6.69	2.57	0.999999994	0.999999994
139.00	31.36	6.74	2.59	0.999999994	0.999999994
140.00	31.59	6.79	2.60	0.999999994	0.999999994
141.00	103.96	5.14	3.70	0.999999994	0.999999994
142.00	116.74	5.11	3.85	0.999999994	0.999999994
143.00	59.92	6.35	3.21	0.999999994	0.999999994
144.00	32.50	6.96	2.66	0.999999994	0.999999994
145.00	32.73	7.01	2.67	0.999999994	0.999999994
146.00	32.96	7.07	2.69	0.999999994	0.999999994
147.00	33.18	7.11	2.70	0.999999994	0.999999994
148.00	33.41	7.16	2.72	0.999999994	0.999999994
149.00	33.64	7.22	2.73	0.999999994	0.999999994
150.00	35.87	7.27	2.75	0.999999994	0.999999994
151.00	106.21	5.60	3.78	0.999999994	0.999999994
152.00	118.96	5.59	3.89	0.999999994	0.999999994
153.00	62.20	6.82	3.35	0.999999994	0.999999994
154.00	34.70	7.44	2.80	0.999999994	0.999999994
155.00	35.01	7.49	2.82	0.999999994	0.999999994
156.00	35.24	7.54	2.83	0.999999994	0.999999994
157.00	35.46	7.59	2.85	0.999999994	0.999999994
158.00	35.69	7.63	2.86	0.999999994	0.999999994
159.00	35.92	7.69	2.88	0.999999994	0.999999994

160.00	56.15	7.74	2.89	0.99999994	0.99999994	
161.00	108.47	6.00	3.84	0.99999994	0.99999994	
162.00	121.22	6.06	3.93	0.99999994	0.99999994	
163.00	60.49	7.29	3.00	0.99999994	0.99999994	
164.00	57.06	7.91	2.94	0.99999994	0.99999994	
165.00	37.29	7.96	2.96	0.99999994	0.99999994	
166.00	37.52	8.01	2.97	0.99999994	0.99999994	
167.00	37.75	8.05	2.99	0.99999994	0.99999994	
168.00	37.98	8.10	3.00	0.99999994	0.99999994	
169.00	38.21	8.15	3.02	0.99999994	0.99999994	
170.00	38.43	8.20	3.03	0.99999994	0.99999994	
171.00	110.72	6.52	3.89	0.99999994	0.99999994	
172.00	123.40	6.53	3.90	0.99999994	0.99999994	
173.00	66.76	7.75	3.61	0.99999994	0.99999994	
174.00	39.35	8.38	3.08	0.99999994	0.99999994	
175.00	39.58	8.42	3.09	0.99999994	0.99999994	
176.00	39.81	8.48	3.11	0.99999994	0.99999994	
177.00	40.03	8.52	3.12	0.99999994	0.99999994	
178.00	40.26	8.57	3.14	0.99999994	0.99999994	
179.00	40.49	8.62	3.15	0.99999994	0.99999994	
180.00	40.72	8.66	3.16	0.99999994	0.99999994	

X AMPLITUDES = 10.00000 10.00000 10.00000 10.00000
X PERIODS = 2.50000 2.50000 2.50000 2.50000
Y AMPLITUDES= 5.00000 5.00000 5.00000 5.00000
Y PERIODS = 2.60000 2.60000 2.60000 2.60000
Z AMPLITUDES= 90.00000 90.00000 0.00000 0.00000
Z PERIODS= 400.00 400.00 400.00 400.00
TIME AT END OF EACH INTERVAL IN SECONDS= 100. 200. 300. 600.
QUANTIZING INTERVAL IN TENTHS OF SECONDS = 3
MINIMUM ANGLE STEP= 1.00000
INITIAL HEADING = 0.
LATITUDE= 45.
SENSOR X,Y,Z SCALE FACTORS = 1.020 1.020 1.020
SENSOR X,Y,Z BIAS IN DEGREES PER HOUR= 1.000 1.000 1.000

TIME(MINUTES)	HEADING	TILT	ERROR	DETCREF	DETCOMP
1.00	73.11	2.86	1.47	0.99999994	0.99999994
2.00	68.82	4.26	1.85	0.99999994	0.99999994
3.00	56.30	3.95	0.93	0.99999994	0.99999994
4.00	13.65	4.19	0.61	0.99999994	0.99999994
5.00	17.85	3.69	0.79	0.99999994	0.99999994
6.00	20.10	1.56	0.89	0.99999994	0.99999994
7.00	20.25	2.37	0.91	0.99999994	0.99999994
8.00	18.00	4.16	0.83	0.99999994	0.99999994
9.00	14.08	5.38	0.68	0.99999994	0.99999994
10.00	9.16	4.70	0.49	0.99999994	0.99999994
11.00	82.61	2.36	1.99	0.99999994	0.99999994
12.00	98.30	3.75	2.37	0.99999994	0.99999994
13.00	45.83	3.03	1.46	0.99999994	0.99999994
14.00	23.20	3.85	1.14	0.99999994	0.99999994
15.00	27.39	3.36	1.32	0.99999994	0.99999994
16.00	29.61	2.16	1.41	0.99999994	0.99999994
17.00	29.73	3.40	1.41	0.99999994	0.99999994
18.00	27.45	5.14	1.32	0.99999994	0.99999994
19.00	23.51	6.26	1.17	0.99999994	0.99999994
20.00	18.62	5.54	0.99	0.99999994	0.99999994
21.00	92.11	2.40	2.50	0.99999994	0.99999994
22.00	107.76	3.65	2.89	0.99999994	0.99999994
23.00	55.35	2.14	1.98	0.99999994	0.99999994
24.00	32.74	3.59	1.66	0.99999994	0.99999994
25.00	36.91	3.20	1.84	0.99999994	0.99999994
26.00	39.11	2.93	1.91	0.99999994	0.99999994
27.00	39.20	4.45	1.91	0.99999994	0.99999994
28.00	36.90	6.17	1.81	0.99999994	0.99999994
29.00	32.94	7.25	1.65	0.99999994	0.99999994
30.00	28.05	6.50	1.46	0.99999994	0.99999994
31.00	101.61	2.98	3.00	0.99999994	0.99999994
32.00	117.26	3.99	3.39	0.99999994	0.99999994
33.00	68.85	1.31	2.49	0.99999994	0.99999994
34.00	42.26	3.45	2.17	0.99999994	0.99999994
35.00	46.43	3.17	2.34	0.99999994	0.99999994
36.00	48.60	3.75	2.41	0.99999994	0.99999994
37.00	48.66	5.43	2.39	0.99999994	0.99999994
38.00	46.36	7.20	2.28	0.99999994	0.99999994
39.00	42.36	8.27	2.12	0.99999994	0.99999994
40.00	37.49	7.52	1.95	0.99999994	0.99999994
41.00	111.15	3.84	3.49	0.99999994	0.99999994

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APPLIED PHYSICS LABORATORY
LAUREL MARYLAND

42.00	126.74	4.60	3.89	0.999999944	0.999999944
43.00	74.36	0.83	2.90	0.999999944	0.999999944
44.00	51.71	3.45	2.67	0.999999944	0.999999944
45.00	35.93	3.31	2.85	0.999999944	0.999999944
46.00	58.09	4.54	2.80	0.999999944	0.999999944
47.00	28.16	6.40	2.80	0.999999944	0.999999944
48.00	55.43	8.22	2.74	0.999999944	0.999999944
49.00	51.84	9.30	2.57	0.999999944	0.999999944
50.00	46.94	8.55	2.40	0.999999944	0.999999944
51.00	120.65	4.83	3.98	0.999999941	0.999999944
52.00	156.24	5.57	4.38	0.999999944	0.999999944
53.00	83.46	1.22	3.45	0.999999944	0.999999944
54.00	61.26	3.58	3.15	0.999999944	0.999999944
55.00	65.44	3.59	3.31	0.999999944	0.999999944
56.00	67.59	5.33	3.35	0.999999941	0.999999944
57.00	67.66	7.32	3.31	0.999999944	0.999999944
58.00	65.32	9.20	3.10	0.999999944	0.999999944
59.00	61.32	10.32	3.00	0.999999944	0.999999944
60.00	56.41	9.57	2.84	0.999999944	0.999999944
61.00	150.19	5.86	4.40	0.999999944	0.999999944
62.00	145.74	6.53	4.07	0.999999944	0.999999944
63.00	93.38	2.03	3.92	0.999999944	0.999999944
64.00	70.78	3.43	3.62	0.999999944	0.999999944
65.00	74.94	3.96	3.77	0.999999944	0.999999944
66.00	77.11	6.10	3.79	0.999999944	0.999999944
67.00	77.18	8.20	3.74	0.999999944	0.999999944
68.00	74.84	10.15	3.60	0.999999944	0.999999944
69.00	70.82	11.29	3.42	0.999999944	0.999999944
70.00	65.90	10.55	3.26	0.999999944	0.999999944
71.00	139.74	6.89	4.93	0.999999944	0.999999944
72.00	155.24	7.58	5.35	0.999999944	0.999999944
73.00	102.90	2.92	4.37	0.999999944	0.999999944
74.00	80.29	4.18	4.07	0.999999944	0.999999944
75.00	84.46	4.43	4.22	0.999999944	0.999999944
76.00	86.64	6.85	4.23	0.999999944	0.999999944
77.00	86.72	9.02	4.16	0.999999944	0.999999944
78.00	84.39	11.01	4.01	0.999999944	0.999999944
79.00	80.36	12.21	3.82	0.999999944	0.999999944
80.00	75.42	11.49	3.67	0.999999944	0.999999944
81.00	149.30	7.90	5.41	0.999999944	0.999999944
82.00	164.74	8.61	5.85	0.999999944	0.999999944
83.00	112.45	3.82	4.82	0.999999944	0.999999944
84.00	89.81	4.58	4.50	0.999999944	0.999999944
85.00	93.94	4.92	4.65	0.999999944	0.999999944
86.00	96.19	7.51	4.65	0.999999944	0.999999944
87.00	96.30	9.78	4.50	0.999999944	0.999999944
88.00	93.97	11.82	4.32	0.999999944	0.999999944
89.00	89.94	13.06	4.22	0.999999944	0.999999944
90.00	84.97	12.37	4.00	0.999999941	0.999999944
91.00	159.86	8.87	5.89	0.999999944	0.999999944
92.00	171.24	9.62	-353.70	0.999999941	0.999999944
93.00	122.01	4.70	5.26	0.999999944	0.999999944
94.00	99.35	5.02	4.93	0.999999944	0.999999944
95.00	103.55	5.42	5.08	0.999999941	0.999999944
96.00	105.70	8.16	5.07	0.999999944	0.999999944
97.00	105.90	10.47	4.99	0.999999944	0.999999944
98.00	103.59	12.55	4.81	0.999999944	0.999999944
99.00	99.50	13.64	4.60	0.999999944	0.999999944
100.00	94.57	13.17	4.44	0.999999944	0.999999944

101.00	168.42	9.79	6.37	0.99999994	0.99999994
102.00	-176.26	10.59	6.77	0.99999994	0.99999994
103.00	151.58	5.56	5.71	0.99999994	0.99999994
104.00	108.90	5.48	5.35	0.99999994	0.99999994
105.00	113.10	5.93	5.50	0.99999994	0.99999994
106.00	115.39	8.74	5.48	0.99999994	0.99999994
107.00	115.54	11.09	5.40	0.99999994	0.99999994
108.00	113.25	13.21	5.21	0.99999994	0.99999994
109.00	109.22	14.53	4.99	0.99999994	0.99999994
110.00	104.21	13.90	4.82	0.99999994	0.99999994
111.00	177.98	10.65	-353.15	0.99999994	0.99999994
112.00	-166.77	11.51	7.23	0.99999994	0.99999994
113.00	141.17	6.38	6.16	0.99999994	0.99999994
114.00	118.48	5.95	5.77	0.99999994	0.99999994
115.00	122.69	6.42	5.92	0.99999994	0.99999994
116.00	125.02	9.27	5.90	0.99999994	0.99999994
117.00	125.22	11.63	5.81	0.99999994	0.99999994
118.00	122.95	13.76	5.62	0.99999994	0.99999994
119.00	118.93	15.14	5.39	0.99999994	0.99999994
120.00	113.89	14.53	5.21	0.99999994	0.99999994
121.00	-172.48	11.45	7.32	0.99999994	0.99999994
122.00	-157.29	12.36	7.68	0.99999994	0.99999994
123.00	150.77	7.16	6.61	0.99999994	0.99999994
124.00	128.08	6.40	6.19	0.99999994	0.99999994
125.00	132.30	6.90	6.35	0.99999994	0.99999994
126.00	134.68	9.74	6.34	0.99999994	0.99999994
127.00	134.92	12.10	6.25	0.99999994	0.99999994
128.00	132.66	14.25	6.05	0.99999994	0.99999994
129.00	128.67	15.65	5.80	0.99999994	0.99999994
130.00	123.61	15.08	5.60	0.99999994	0.99999994
131.00	-162.95	12.16	7.79	0.99999994	0.99999994
132.00	-147.81	13.14	8.11	0.99999994	0.99999994
133.00	160.37	7.89	7.08	0.99999994	0.99999994
134.00	137.69	6.83	6.62	0.99999994	0.99999994
135.00	141.93	7.34	6.79	0.99999994	0.99999994
136.00	144.36	10.14	6.78	0.99999994	0.99999994
137.00	144.63	12.47	6.70	0.99999994	0.99999994
138.00	142.44	14.63	6.50	0.99999994	0.99999994
139.00	138.45	16.06	6.24	0.99999994	0.99999994
140.00	133.36	15.55	6.02	0.99999994	0.99999994
141.00	-153.43	12.80	8.25	0.99999994	0.99999994
142.00	-138.34	13.84	8.52	0.99999994	0.99999994
143.00	169.97	8.57	7.55	0.99999994	0.99999994
144.00	147.32	7.24	7.06	0.99999994	0.99999994
145.00	151.56	7.75	7.24	0.99999994	0.99999994
146.00	154.04	10.47	7.25	0.99999994	0.99999994
147.00	154.36	12.77	7.17	0.99999994	0.99999994
148.00	152.21	14.92	6.98	0.99999994	0.99999994
149.00	148.24	16.36	6.71	0.99999994	0.99999994
150.00	143.14	15.88	6.47	0.99999994	0.99999994
151.00	-143.92	13.36	8.69	0.99999994	0.99999994
152.00	-128.86	14.45	8.92	0.99999994	0.99999994
153.00	179.56	9.19	-351.97	0.99999994	0.99999994
154.00	156.95	7.62	7.52	0.99999994	0.99999994
155.00	161.20	8.11	7.71	0.99999994	0.99999994
156.00	163.73	10.73	7.74	0.99999994	0.99999994
157.00	164.10	12.97	7.68	0.99999994	0.99999994
158.00	161.99	15.10	7.48	0.99999994	0.99999994
159.00	158.05	16.56	7.21	0.99999994	0.99999994

160.00	152.94	16.12	6.96	0.99999994	0.99999994
161.00	-139.42	13.82	9.11	0.99999994	0.99999994
162.00	-119.37	14.97	9.29	0.99999994	0.99999994
163.00	-170.97	9.74	8.51	0.99999994	0.99999994
164.00	166.50	7.95	8.00	0.99999994	0.99999994
165.00	170.85	8.45	8.19	0.99999994	0.99999994
166.00	173.41	10.91	-351.76	0.99999994	0.99999994
167.00	173.82	13.08	-351.80	0.99999994	0.99999994
168.00	171.75	15.10	8.02	0.99999994	0.99999994
169.00	167.85	16.65	7.75	0.99999994	0.99999994
170.00	162.74	16.26	7.48	0.99999994	0.99999994
171.00	-124.92	14.19	9.51	0.99999994	0.99999994
172.00	-109.87	15.39	9.64	0.99999994	0.99999994
173.00	-101.31	10.23	8.99	0.99999994	0.99999994
174.00	176.20	8.25	-351.51	0.99999994	0.99999994
175.00	-179.54	8.70	8.69	0.99999994	0.99999994
176.00	-176.92	11.02	8.76	0.99999994	0.99999994
177.00	-174.48	13.10	8.74	0.99999994	0.99999994
178.00	-176.50	15.15	4.58	0.99999994	0.99999994
179.00	177.64	16.64	-351.64	0.99999994	0.99999994
180.00	172.53	16.29	-351.97	0.99999994	0.99999994

X AMPLITUDES = 4.00000 4.00000 4.00000 4.00000
X PERIODS = 1.50000 1.50000 1.50000 1.50000
Y AMPLITUDES= 3.00000 3.00000 3.00000 3.00000
Y PERIODS = 1.60000 1.60000 1.60000 1.60000
Z AMPLITUDES= 90.00000 90.00000 0.00000 0.00000
Z PERIODS= 400.00 400.00 400.00 400.00
TIME AT END OF EACH INTERVAL IN SECONDS= 100. 200. 300. 400.
QUANTIZING INTERVAL IN TENTHS OF SECONDS = 2
MINIMUM ANGLE STEP= 0.50000
INITIAL HEADING = 0.
LATITUDE= 45.
SENSOR X,Y,Z SCALE FACTORS = 1.020 1.020 1.020
SENSOR X,Y,Z BIAS IN DEGREES PER HOUR= 1.000 1.000 1.000

TIME(MINUTES)	HEADING	TILT	ERROR	DETCREF	DETCOMP
1.00	74.88	0.57	1.55	0.99999994	0.99999994
2.00	85.48	0.54	1.73	0.99999994	0.99999994
3.00	29.95	1.28	0.68	0.99999994	0.99999994
4.00	-0.01	1.17	0.05	0.99999994	0.99999994
5.00	2.17	0.98	0.15	0.99999994	0.99999994
6.00	-0.01	0.73	0.08	0.99999994	0.99999994
7.00	2.16	0.89	0.18	0.99999994	0.99999994
8.00	-0.01	0.86	0.10	0.99999994	0.99999994
9.00	2.16	1.31	0.20	0.99999994	0.99999994
10.00	-0.02	1.42	0.13	0.99999994	0.99999994
11.00	74.87	1.18	1.67	0.99999994	0.99999994
12.00	85.46	1.38	1.85	0.99999994	0.99999994
13.00	29.94	1.44	0.81	0.99999994	0.99999994
14.00	-0.00	1.43	0.18	0.99999994	0.99999994
15.00	2.16	1.74	0.28	0.99999994	0.99999994
16.00	-0.01	1.73	0.20	0.99999994	0.99999994
17.00	2.15	2.16	0.30	0.99999994	0.99999994
18.00	-0.03	2.23	0.23	0.99999994	0.99999994
19.00	2.13	2.71	0.42	0.99999994	0.99999994
20.00	-0.06	2.84	0.25	0.99999994	0.99999994
21.00	74.86	2.55	1.78	0.99999994	0.99999994
22.00	85.46	2.71	1.96	0.99999994	0.99999994
23.00	29.91	2.56	0.93	0.99999994	0.99999994
24.00	-0.01	2.59	0.31	0.99999994	0.99999994
25.00	2.15	3.02	0.41	0.99999994	0.99999994
26.00	-0.04	3.08	0.33	0.99999994	0.99999994
27.00	2.12	3.54	0.42	0.99999994	0.99999994
28.00	-0.07	3.64	0.35	0.99999994	0.99999994
29.00	2.08	4.12	0.44	0.99999994	0.99999994
30.00	-0.11	4.25	0.37	0.99999994	0.99999994
31.00	74.85	3.95	1.88	0.99999994	0.99999994
32.00	85.47	4.11	2.07	0.99999994	0.99999994
33.00	29.87	3.98	1.03	0.99999994	0.99999994
34.00	-0.03	3.93	0.43	0.99999994	0.99999994
35.00	2.12	4.38	0.53	0.99999994	0.99999994
36.00	-0.07	4.47	0.45	0.99999994	0.99999994
37.00	2.07	4.94	0.54	0.99999994	0.99999994
38.00	-0.12	5.05	0.47	0.99999994	0.99999994
39.00	2.03	5.54	0.56	0.99999994	0.99999994
40.00	-0.17	5.67	0.48	0.99999994	0.99999994
41.00	74.85	5.36	1.97	0.99999994	0.99999994

42.00	85.49	5.52	2.10	0.9999	0.9999
43.00	29.41	5.25	1.13	0.9999	0.9999
44.00	-0.06	5.41	0.55	0.9999	0.9999
45.00	2.07	5.17	0.66	0.9999	0.9999
46.00	-0.12	5.87	0.57	0.9999	0.9999
47.00	2.02	6.35	0.66	0.9999	0.9999
48.00	-0.18	6.47	0.58	0.9999	0.9999
49.00	1.96	6.96	0.67	0.9999	0.9999
50.00	-0.24	7.09	0.59	0.9999	0.9999
51.00	74.86	6.78	2.04	0.9999	0.9999
52.00	85.52	6.93	2.24	0.9999	0.9999
53.00	29.74	6.68	1.22	0.9999	0.9999
54.00	-0.11	6.70	0.67	0.9999	0.9999
55.00	2.02	7.17	0.76	0.9999	0.9999
56.00	-0.18	7.24	0.68	0.9999	0.9999
57.00	1.95	7.76	0.77	0.9999	0.9999
58.00	-0.25	7.84	0.69	0.9999	0.9999
59.00	1.87	8.34	0.78	0.9999	0.9999
60.00	-0.32	8.51	0.70	0.9999	0.9999
61.00	74.86	8.19	2.11	0.9999	0.9999
62.00	85.57	8.35	2.31	0.9999	0.9999
63.00	29.64	8.04	1.29	0.9999	0.9999
64.00	-0.17	8.10	0.76	0.9999	0.9999
65.00	1.95	8.58	0.87	0.9999	0.9999
66.00	-0.25	8.69	0.79	0.9999	0.9999
67.00	1.86	9.18	0.87	0.9999	0.9999
68.00	-0.33	9.30	0.80	0.9999	0.9999
69.00	1.77	9.79	0.86	0.9999	0.9999
70.00	-0.42	9.93	0.81	0.9999	0.9999
71.00	74.88	9.61	2.16	0.9999	0.9999
72.00	85.62	9.76	2.36	0.9999	0.9999
73.00	29.53	9.45	1.36	0.9999	0.9999
74.00	-0.24	9.51	0.89	0.9999	0.9999
75.00	1.86	9.99	0.97	0.9999	0.9999
76.00	-0.33	10.11	0.90	0.9999	0.9999
77.00	1.76	10.59	0.98	0.9999	0.9999
78.00	-0.43	10.72	0.90	0.9999	0.9999
79.00	1.66	11.21	0.98	0.9999	0.9999
80.00	-0.53	11.34	0.91	0.9999	0.9999
81.00	74.89	11.02	2.19	0.9999	0.9999
82.00	85.68	11.18	2.41	0.9999	0.9999
83.00	29.41	10.85	1.42	0.9999	0.9999
84.00	-0.52	10.92	1.10	0.9999	0.9999
85.00	1.76	11.40	1.07	0.9999	0.9999
86.00	-0.43	11.52	1.00	0.9999	0.9999
87.00	1.65	12.01	1.07	0.9999	0.9999
88.00	-0.54	12.14	1.00	0.9999	0.9999
89.00	1.54	12.63	1.08	0.9999	0.9999
90.00	-0.65	12.76	1.01	0.9999	0.9999
91.00	74.92	12.44	2.21	0.9999	0.9999
92.00	85.76	12.59	2.44	0.9999	0.9999
93.00	29.26	12.27	1.47	0.9999	0.9999
94.00	-0.42	12.31	1.10	0.9999	0.9999
95.00	1.65	12.82	1.17	0.9999	0.9999
96.00	-0.54	12.94	1.10	0.9999	0.9999
97.00	1.53	13.43	1.17	0.9999	0.9999
98.00	-0.66	13.55	1.10	0.9999	0.9999
99.00	1.40	14.05	1.17	0.9999	0.9999
100.00	-0.79	14.14	1.10	0.9999	0.9999

101.00	74.94	13.86	2.22	0.99999994	0.99999994
102.00	85.84	14.01	2.46	0.99999994	0.99999994
103.00	29.10	13.68	1.51	0.99999994	0.99999994
104.00	-0.53	13.75	1.20	0.99999994	0.99999994
105.00	1.53	14.23	1.26	0.99999994	0.99999994
106.00	-0.66	14.36	1.20	0.99999994	0.99999994
107.00	1.39	14.84	1.26	0.99999994	0.99999994
108.00	-0.80	14.97	1.20	0.99999994	0.99999994
109.00	1.25	15.46	1.26	0.99999994	0.99999994
110.00	-0.94	15.60	1.20	0.99999994	0.99999994
111.00	74.98	15.27	2.21	0.99999994	0.99999994
112.00	85.94	15.43	2.46	0.99999994	0.99999994
113.00	28.92	15.09	1.54	0.99999994	0.99999994
114.00	-0.65	15.16	1.29	0.99999994	0.99999994
115.00	1.39	15.65	1.35	0.99999994	0.99999994
116.00	-0.79	15.77	1.29	0.99999994	0.99999994
117.00	1.24	16.26	1.35	0.99999994	0.99999994
118.00	-0.94	16.39	1.29	0.99999994	0.99999994
119.00	1.09	16.88	1.34	0.99999994	0.99999994
120.00	-1.10	17.02	1.28	0.99999994	0.99999994
121.00	75.02	16.69	2.18	0.99999994	0.99999994
122.00	86.05	16.85	2.45	0.99999994	0.99999994
123.00	28.72	16.51	1.56	0.99999994	0.99999994
124.00	-0.78	16.58	1.36	0.99999994	0.99999994
125.00	1.24	17.06	1.44	0.99999994	0.99999994
126.00	-0.94	17.19	1.38	0.99999994	0.99999994
127.00	1.08	17.68	1.43	0.99999994	0.99999994
128.00	-1.10	17.81	1.37	0.99999994	0.99999994
129.00	0.91	18.30	1.42	0.99999994	0.99999994
130.00	-1.27	18.44	1.37	0.99999994	0.99999994
131.00	75.06	18.11	2.14	0.99999994	0.99999994
132.00	86.17	18.26	2.42	0.99999994	0.99999994
133.00	28.50	17.92	1.57	0.99999994	0.99999994
134.00	-0.93	17.99	1.47	0.99999994	0.99999994
135.00	1.08	18.48	1.52	0.99999994	0.99999994
136.00	-1.10	18.61	1.46	0.99999994	0.99999994
137.00	0.90	19.09	1.51	0.99999994	0.99999994
138.00	-1.27	19.22	1.46	0.99999994	0.99999994
139.00	0.72	19.72	1.50	0.99999994	0.99999994
140.00	-1.45	19.85	1.45	0.99999994	0.99999994
141.00	75.11	19.53	2.09	0.99999994	0.99999994
142.00	86.51	19.68	2.38	0.99999994	0.99999994
143.00	28.26	19.34	1.57	0.99999994	0.99999994
144.00	-1.09	19.41	1.55	0.99999994	0.99999994
145.00	0.90	19.90	1.59	0.99999994	0.99999994
146.00	-1.27	20.02	1.54	0.99999994	0.99999994
147.00	0.71	20.51	1.59	0.99999994	0.99999994
148.00	-1.46	20.64	1.54	0.99999994	0.99999994
149.00	0.52	21.14	1.58	0.99999994	0.99999994
150.00	-1.65	21.27	1.53	0.99999994	0.99999994
151.00	75.16	20.95	2.02	0.99999994	0.99999994
152.00	86.45	21.10	2.33	0.99999994	0.99999994
153.00	28.00	20.75	1.57	0.99999994	0.99999994
154.00	-1.26	20.82	1.63	0.99999994	0.99999994
155.00	0.71	21.31	1.67	0.99999994	0.99999994
156.00	-1.45	21.44	1.62	0.99999994	0.99999994
157.00	0.51	21.93	1.66	0.99999994	0.99999994
158.00	-1.65	22.06	1.61	0.99999994	0.99999994
159.00	0.30	22.55	1.65	0.99999994	0.99999994

160.00	-1.86	22.69	1.81	0.99999994	0.99999994
161.00	75.22	22.36	1.93	0.99999994	0.99999994
162.00	86.62	22.52	2.26	0.99999994	0.99999994
163.00	27.72	22.17	1.56	0.99999994	0.99999994
164.00	-1.44	22.24	1.71	0.99999994	0.99999994
165.00	0.50	22.73	1.74	0.99999994	0.99999994
166.00	-1.65	22.86	1.70	0.99999994	0.99999994
167.00	0.29	23.35	1.73	0.99999994	0.99999994
168.00	-1.86	23.48	1.69	0.99999994	0.99999994
169.00	0.07	23.97	1.72	0.99999994	0.99999994
170.00	-2.08	24.11	1.68	0.99999994	0.99999994
171.00	75.29	23.76	1.82	0.99999994	0.99999994
172.00	86.79	23.93	2.17	0.99999994	0.99999994
173.00	27.43	23.58	1.54	0.99999994	0.99999994
174.00	-1.63	23.66	1.76	0.99999994	0.99999994
175.00	0.29	24.15	1.81	0.99999994	0.99999994
176.00	-1.86	24.27	1.77	0.99999994	0.99999994
177.00	0.06	24.76	1.79	0.99999994	0.99999994
178.00	-2.08	24.90	1.76	0.99999994	0.99999994
179.00	-6.18	25.39	1.78	0.99999994	0.99999994
180.00	-2.32	25.53	1.75	0.99999994	0.99999994

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FLUIDIC HEADING SYSTEM STUDY: SUMMARY REPORT.(U)
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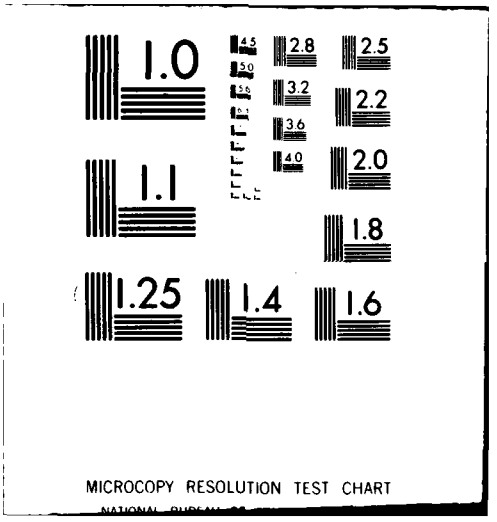
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MICROCOPY RESOLUTION TEST CHART

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APPENDIX G.--CHECK ON ACCURACY OF REFERENCE ALGORITHM

In the study of computation algorithms (Section 4), the "error" is simply the difference between a particular algorithm's output and that of the "reference" algorithm. A nagging question remains as to the sufficiency of the reference algorithm. A precise mathematical solution for any but extremely simple input motion is beyond reach.

One source of confidence is, of course, the small errors between the reference and the best of the test algorithms.

An additional check on the reference algorithm was made utilizing the mathematical formulation of attitude as a result of coning motion. This formulation is found in the literature[1]. Coning motion is the result of quadrature motion on two of the axes, a motion which presents a most difficult test of computation adequacy.

For X and Y axis motion given by

$$\dot{x} = A \sin Bt$$

$$\dot{y} = A \cos Bt$$

$$R^2 = A^2 + B^2 \quad ,$$

the C-matrix components C_{11} and C_{21} at any time t may be expressed as

$$C_{11} = \frac{1}{2R} [(B+R) \cos (B-R)t - (B-R) \cos (B+R)t]$$

$$C_{21} = \frac{A^2}{R^2} \sin Bt + \frac{B}{2R^2} [(B-R) \sin (B+R)t + (B+R)\sin(B-R)t] \quad .$$

A suitable choice of A and B which makes the arithmetic simple, while giving a severe but practical test is $A = 0.15\pi$ and $B = 0.2\pi$. Then C_{11} and C_{12} reduce to

$$C_{11} = 0.9 \cos 0.05\ \pi t + 0.2 \cos 0.45\ \pi t$$

---

[1] A.A. Sutherland, Jr. and P. O. Bongiovanni, "Data Smoothing for Enhancement of Strapdown Inertial System Accuracy: Interpolative-Predictive Algorithms," Analytic Sciences Corp. Report TR-155-1, 31 August 1969.

$$C_{21} = 0.36 \sin 0.2\,\pi t - 0.08 \sin 0.45\,\pi t$$

$$- 0.72 \sin 0.05\,\pi t$$

The period of this motion is 40 s. At t = 10 s, $C_{11} = 0$ for a heading of 90 deg, while at t = 20, $C_{21} = 0$ and the heading is 180 deg.

This motion was used to test the "reference" algorithm and confirmed precise agreement with the theoretical results.

NOTE

Since the program was written in terms of angle rather than rate, the rates $\dot{x}$ and $\dot{y}$ must be integrated to obtain the proper input.

$$x = -\frac{A}{B}\cos Bt + \frac{A}{B} .$$

$$y = \frac{A}{B}\sin Bt .$$

The constant of integration $\frac{A}{B}$ must be added to x so that x(0) = 0, since the expressions for the C-matrix elements assume initial alignment (identify matrix) at the start. It is, in fact, this initial alignment which makes the equations complex, since coning takes place about an axis offset from the vertical.

As a matter of interest, the tilt angle θ of the Z axis is given by

$$\cos\theta = C_{33} = 1 - \frac{A^2}{R^2}(1 - \cos Rt)$$

$$= 0.64 + 0.36 \cos 0.25\,\pi t$$

This describes conical motion with $\theta_{max} = 74$ deg and a period of 8 s. Hence, five complete cones are swept out by the Z axis during the 40 seconds it takes for the heading to make one complete revolution.

APPENDIX H.--PROGRAM LISTING FOR SECTION 4

The results of Section 4 were obtained with the following computer programs:

a) TNKSIM calculates the reference heading and tilt angles and develops the record of angle increments which would be measured by the vehicle-mounted sensors. Earth rotation is accounted for in these outputs.

b) TNKTST models the anticipated microprocessor-based system, including a pulse-balanced integrator as the analog-to-digital device.

```
04:25:20    FORTRAN       SYSTEMS REAL-TIME MONITOR-7.0

S E L  E X T E N D E D  F O R T R A N  I V  (R E V - 0 / 7 8 S E P 0 8)

TNKSIM

                    PROGRAM TNKSIM
                    IMPLICIT REAL*8(A-H,O-Z)
                    REAL*4 SDOUT,SHDR,SCVN
                    DIMENSION AMP(3),FREQ(3),CVN(3,3),CVNN(3,3),PPNV(3),ERATE(3),
                  1 HDR(20),PNV(3),DPNV(3),DPNVM(3,3),ERIN(3),DOUT(5,250),
                  2 SDOUT(2500),SHDR(40),CVNS(9)
                    DIMENSION DPNVNS(3,3),DPNVI(3)
                    DIMENSION CVNR(3),CVNC(3),CVNP(6)
                    DIMENSION DAMP(3),DFREQ(3),KX(3)
                    EQUIVALENCE (SHDR(1),HDR(1)),(DOUT(1,1),SDOUT(1))
                    EQUIVALENCE (CVNR(1),CVNP(1)),(CVNC(1),CVNP(4)),(CVN(1,1),
                  1             CVNS(1))
                    PI=3.14159265360+0
                    KTY=5
                    TSIM1=0
                    WRITE(KTY,600)
              600   FORMAT(' ENTER 1 TO CONTINUE PREVIOUS RUN, 0 TO START NEW RUN'/)
                    READ(KTY,201)KB
                    IF(KB.EQ.0)GO TO 103
            C
            C  FIND END OF PREVIOUS DATA AND GET C MARRIX
            C
              605   READ(21)SHDR
                    TSIM1=HDR(12)
                    WRITE(KTY,601)HDR(12)
              601   FORMAT(' PRESENT RUNNING TIME = ',F7.0,2X,'SECONDS'/
                  1 ' ENTER 1 TO START SIMULATION, 0 TO SEARCH FURTHER'/)
                    READ(KTY,201)KA
                    READ(21)CVNS
              603   READ(21,END=602)SDOUT
                    GO TO 603
              602   READ(21)CVNS
                    READ(21,END=604)SDOUT
              604   IF(KA.EQ.0)GO TO 605
                    PAUSE A
            C
            C       ENTER CONSTANTS FOR INPUT MOTION
            C
              103   WRITE(KTY,1)
                  1 FORMAT(' ENTER X AMPLITUDE,PERIOD,PHASE: 1=SIN,0=COS'/)
                    READ(KTY,2)AMP(1),FREQ(1),KX(1)
                  2 FORMAT(2D20.12,I1)
                    WRITE(KTY,3)
                  3 FORMAT(' ENTER Y AMPLITUDE,PERIOD,PHASE;1=SIN,0=COS'/)
                    READ(KTY,4)AMP(2),FREQ(2),KX(2)
                  4 FORMAT(2D20.12,I1)
                    WRITE(KTY,5)
                  5 FORMAT(' ENTER Z AMPLITUDE,PERIOD,PHASE;1=SIN,0=COS'/)
                    READ(KTY,6)AMP(3),FREQ(3),KX(3)
                  6 FORMAT(2D20.12,I1)
                    WRITE(KTY,7)
                  7 FORMAT(' ENTER SAMPLING INTERVAL IN SECONDS'/)
                    READ(KTY,8)DTSIM
                  8 FORMAT(D20.12)
```

04:25:20 FORTRAN SYSTEMS REAL-TIME MONITOR-7.0

S E L E X T E N D E D F O R T R A N I V (R E V - 0 / 7 8 S E P 0 8)

TNKSIM

```
      WRITE(KTY,9)
    9 FORMAT(' ENTER LATITUDE IN DEGREES'/)
      READ(KTY,91)LAT
   91 FORMAT(I2)
      IF(KB.EQ.0)GO TO 104
      WRITE(KTY,105)TSIM
  105 FORMAT(' PRESENT RUNNING TIME = ',F9.1,1X,'SECONDS'/
     1 ' ENTER CONTINUE RUNNING TIME IN SECONDS'/)
      GO TO 107
  104 WRITE(KTY,92)
   92 FORMAT(' ENTER RUNNING TIME IN SECONDS'/)
  107 READ(KTY,93)TMAX
   93 FORMAT(D20.12)
      WRITE(KTY,94)
   94 FORMAT(' ENTER PRINT TIME INTERVAL IN SECONDS'/)
      READ(KTY,71)DPTIME
   71 FORMAT(D20.12)
      WRITE(KTY,200)
  200 FORMAT(' ENTER 1 TO INPUT VEHICLE AXIS MOTION'/)
      READ(KTY,201)KV
  201 FORMAT(I1)
      PTIME=DPTIME+TSIM1
      LPCNT=25
   96 FORMAT(1X,'LAT',3X,'SAMPLE',10X,'AMPLITUDE',18X,
     1 'PERIOD',13X,'PHASE(0=COS,1=SIN)',10X,'MOTION(1=V,0=N)'/)
   95 FORMAT(I4,7F9.3,3I5,20X,I1//)
   97 FORMAT(5X,'TIME',7X,'C11',8X,'C12',8X,'C13',8X,
     1 'C21',8X,'C22',8X,'C23',8X,'C31',8X,'C32',8X,
     2 'C33',6X,'HEADING',4X,'TILT'/)
C
C     INITIALIZE C MATRIX
C
      DO 106 K=1,3
  106 DAMP(K)=AMP(K)
      IF(KB.EQ.1)GO TO 102
      DO 10 K=1,3
      DO 10 J=1,3
      IF(K.EQ.J)GO TO 101
      CVN(J,K)=0.0D+0
      GO TO 10
  101 CVN(J,K)=1.0D+0
   10 CONTINUE
C
C     INITIALIZE ANGULAR INPUT MOTION AND EARTH ROTATION MATRIX
C
  102 EDOT=PI/43200.0D+0
      TSIM=TSIM1
      DO 11 K=1,3
      IF(KX(K).EQ.0)GO TO 112
      PPNV(K)=0.0
      GO TO 11
  112 PPNV(K)=DAMP(K)
   11 CONTINUE
      SLAT=LAT*PI/180.0D+0
```

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TMKSIM

```
                      EKATE(1)=DCOS(SLAT)*EDOT                                  0113.000
                      EKATE(2)=0.0D+0                                           0114.000
                      EKATE(3)=-DSIN(SLAT)*EDOT                                 0115.000
                      HDR(1)=DTSIM                                              0116.000
                      DO 111 K=1,3                                              0117.000
                      HDR(1+K)=AMP(K)                                           0118.000
                      HDR(4+K)=FREQ(K)                                          0119.000
                      DFREQ(K)=2.0D+0*PI/FREQ(K)                                0120.000
                       HDR(14+K)=KX(K)                                          0121.000
                      HDR(18)=KV                                                0122.000
                111   HDR(7+K)=EKATE(K)                                         0123.000
                      HDR(19)=KB                                                0124.000
                      HDR(11)=SLAT                                              0125.000
                      HDR(12) = TMAX                                            0126.000
                      WRITE(21)SHDR                                             0127.000
                      DOUT(4,1)=0.0D+0                                          0128.000
                      WRITE(21)CVNS                                             0129.000
                C                                                               0130.000
                C     TOP OF PROCESSING LOOP                                    0131.000
                C     OBTAIN INCREMENTS OF INPUT ANGLES                         0132.000
                C                                                               0133.000
                119   DO 15 K=1,250                                             0134.000
                      TSIM=TSIM+DTSIM                                           0135.000
                      DO 16 K=1,3                                               0136.000
                      THETA=DFREQ(K)*(TSIM-TSIM1)                               0137.000
                      IF(KX(K).EQ.0)GO TO 122                                   0138.000
                      PNV(K)=DAMP(K)*DSIN(THETA)                                0139.000
                      GO TO 120                                                 0140.000
                122   PNV(K)=DAMP(K)*DCOS(THETA)                                0141.000
                120   DPNVI(K)=PNV(K)-PPNV(K)                                   0142.000
                 16   PPNV(K)=PNV(K)                                            0143.000
                      IF(KV.EQ.0)GO TO 164                                      0144.000
                      DO 165 K=1,3                                              0145.000
                165   DPNV(K)=DPNVI(K)                                          0146.000
                      GO TO 166                                                 0147.000
                C     ROTATE INTO VEHICLE AXIS                                  0148.000
                C                                                               0149.000
                164   DO 163 J=1,3                                              0150.000
                      DPNV(J)=0.0D+0                                            0151.000
                      DO 163 K=1,3                                              0152.000
                163   DPNV(J)=DPNV(J)+CVN(K,J)*DPNVI(K)                         0153.000
                C                                                               0154.000
                C     DETERMINE INCREMENT MATRIX AND ROTATE C MATRIX            0155.000
                C                                                               0156.000
                166   DPNVS=0.0D+0                                              0157.000
                      DPNVS(2,1)=DPNV(3)                                        0158.000
                      DPNVS(1,2)=-DPNV(3)                                       0159.000
                      DPNVS(1,3)=DPNV(2)                                        0160.000
                      DPNVS(3,1)=-DPNV(2)                                       0161.000
                      DPNVS(3,2)=DPNV(1)                                        0162.000
                      DPNVS(2,3)=-DPNV(1)                                       0163.000
                      DO 161 J=1,3                                              0164.000
                      DO 161 K=1,3                                              0165.000
                      DPNVMS(J,K)=0.0D+0                                        0166.000
```

S E L E X T E N D E D F O R T R A N I V (R E V - 0 / 7 8 S E P 0 6)

TNKSIM

```
      DO 162 L=1,3
  162 DPNVMS(J,K)=DPNVMS(J,K)+DPNVM(J,L)*0.5*DPNVM(L,K)
  161 CONTINUE
      DO 12 J=1,3
      DO 12 K=1,3
      TEMP=0.0D+0
      DO 121 L=1,3
  121 TEMP=TEMP+CVN(J,L)*(DPNVM(L,K)+DPNVMS(L,K))
   12 CVNN(J,K)=CVN(J,K)+TEMP
      DO 13 J=1,3
      DO 13 K=1,3
   13 CVN(J,K)=CVNN(J,K)
C
C     DETERMINE SECOND EARTH ROTATION ON VEHICLE AXIS
C
      DO 14 J=1,3
      ERIN(J)=0.0D+0
      DO 141 K=1,3
  141 ERIN(J)=ERIN(J)+CVN(K,J)*ERATE(K)
   14 DOUT(J,N)=DPNV(J)+ERIN(J)*DTSIM
      DOUT(4,N)=DATAN2(CVN(2,1),CVN(1,1))*180.0D+0/PI
      SCVN=CVN(3,3)
      DOUT(5,N)=ACOS(SCVN)*180.0/PI
      IF(TSIM.LT.(PTIME-1.0D-4))GO TO 15
      PTIME=PTIME+DPTIME
      IF(LPCNT.LT.25)GO TO 151
      LPCNT=0
      WRITE(7,152)
  152 FORMAT('1')
      WRITE(7,96)
      WRITE(7,95)LAT,DTSIM,(AMP(K),K=1,3),(FREQ(J),J=1,3)
     1          ,(KX(L),L=1,3),KV
      WRITE(7,97)
  151  WRITE(7,133)TSIM,     ((CVN(J,K),K=1,3),J=1,3),DOUT(4,N)
     1            ,DOUT(5,N)
  133 FORMAT(1X,F9.3,9F11.6,F11.4,F9.3)
      DO 135 J=1,3
      TEMP=0.0
      DO 136 K=1,3
  136 TEMP=TEMP+CVN(J,K)**2
  135 CVAR(J)=DSQRT(TEMP)
      DO 137 K=1,3
      TEMP=0.0
      DO 138 J=1,3
  138 TEMP=TEMP+CVN(J,K)**2
  137 CVNC(K)=DSQRT(TEMP)
      CVND=CVN(1,1)*CVN(2,2)*CVN(3,3)+
     1     CVN(1,3)*CVN(2,1)*CVN(3,2)+
     2     CVN(1,2)*CVN(2,3)*CVN(3,1)-
     3     CVN(1,3)*CVN(2,2)*CVN(3,1)-
     4     CVN(1,2)*CVN(2,1)*CVN(3,3)-
     5     CVN(1,1)*CVN(2,3)*CVN(3,2)
      WRITE(7,139)CVND,(CVNP(J),J=1,6)
  139 FORMAT(5X,7F13.8)
```

```
04:25:20    FORTRAN       SYSTEMS REAL-TIME MONITOR-7.0

S E L  E X T E N D E D  F O R T R A N  I V  (R E V - 0 / 7 8 S E P 0 8)

TNKSIM

                        LPCNT=LPCNT+1                                          0221.000
                   15   CONTINUE                                               0222.000
                   21   WRITE(21)SOUOT                                         0223.000
                        IF((TSIM+.001).LT.TMAX)GO TO 119                       0224.000
                        WRITE(7,152)                                           0225.000
                        ENDFILE 21                                             0226.000
                        WRITE(21)CVNS                                          0227.000
                        ENDFILE 21                                             0228.000
                        WRITE(KTY,81)                                          0229.000
                   81   FORMAT(' ENTER 1 TO CONTINUE RUN, 0 TO EXIT'/)         0230.000
                        READ(KTY,201)KB                                        0231.000
                        IF(KB.EQ.0)GO TO 20                                    0232.000
                        TSIM=TMAX                                              0233.000
                        GO TO 103                                              0234.000
                   20   REWIND 21                                              0235.000
                        END                                                    0236.000
```

```
04:24:40    FORTRAN       SYSTEMS REAL-TIME MONITOR-7.0

S E L  E X T E N D E D  F O R T R A N  I V  (R E V - 0 / 7 8 S E P 0 8)

TNKTST

                    PROGRAM TNKTST
                    IMPLICIT INTEGER(A-C,E-Z)
                    IMPLICIT REAL*8(D)
                    REAL*4 SHDR,SIND,SDDHDR,SDDUD,SCVN,SHDRC
                    DIMENSION SHDR(40),DHDR(20),ERATE(3),DDHDR(20),SDDHDR(40),
                  1 ERIN(3),CVN(3,3),SIND(2500),DIND(5,250),DPIV(3),DDPIV(3),
                  3 PIVI(3),PINI(3),PNVI(3),PNVC(3,3),CVND(3,3),DDUD(4,500),
                  3 SDDUD(4000),DCVN(3,3),DCVNS(9),DCVNS1(3,3),CVNS(9),
                  4  SHDRC(40),DHDRC(20)
                    DIMENSION DCVNR(3),DCVNC(3),DCVNP(6),PNVM(3,3)
                    DIMENSION DINTD(3),CINT(3),PNVMS(3,3)
                    EQUIVALENCE (DCVNR(1),DCVNP(1)),(DCVNC(1),DCVNP(4))
                    EQUIVALENCE (DDUD(1,1),SDDUD(1)),(SIND(1),DIND(1,1)),
                  1 (SHDR(1),DHDR(1)),(DDHDR(1),SDDHDR(1)),
                  2 (DCVNS(1),DCVNS1(1,1)),(CVN(1,1),CVNS(1)),
                  3 (SHDRC(1),DHDRC(1))
                    DATA ONE/Z40000000/
                    DPI=3.14159265360+0
                    KTY=5
            C
            C       ENTER CONSTANTS
            C
             105    READ(21)SHDR
                    WRITE(KTY,1)DHDR(1)
                1   FORMAT(' INPUT SAMPLING TIME = ',F10.4,2X,'SECONDS' )
                    WRITE(KTY,82)DHDR(12)
              82    FORMAT(' RUNNING TIME = ',F10.4,2X,'SECONDS' )
                    WRITE(KTY,7)
              7     FORMAT(' ENTER INTEGRATOR THRESHOLD N : VALUE=2**(-N) RAD.'/)
                    READ (KTY,2)BIT
              2     FORMAT(I2)
                    WRITE(KTY,83)
              83     FORMAT(' ENTER RUNNING TIME IN SECONDS'/)
                    READ(KTY,84)DTRUN
              84    FORMAT(D20.12)
                    WRITE(KTY,8)
              8     FORMAT(' ENTER PRINT TIME INTERVAL IN SECONDS'/)
                    READ(KTY,81)DPTIME
              81    FORMAT(D20.12)
                    WRITE(KTY,11)
              11    FORMAT(' ENTER PLOT TIME INTERVAL IN SECONDS'/)
                    READ(KTY,81)DPLTME
                    WRITE(KTY,9)
                9   FORMAT(' ENTER NO. OF BITS FOR PROCESSING'/)
                    READ(KTY,2)MBIT
                    KBIT=MBIT-1
                    SBIT=32-KBIT
                    WRITE(KTY,85)
              85    FORMAT(' ENTER 1 FOR 2ND ORDER UPDATING OF C MATRIX'/)
                    READ(KTY,2)KU
            C
            C       SET SCALE FACTORS AND COMPUTE EARTH RATE MATRIX
            C
                    DEDOT=DPI/43200.0D+0
```

SEL EXTENDED FORTRAN IV (REV-0/78SEP06)

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TNKTST

```
                      DRATE=1.0D+0/(2**KBIT)                                    0055.000
                      DANG=DRATE                                                0056.000
                      DCSC=1.0D+0/(2**(KBIT-1))                                 0057.000
                      DTIME=1.0D+0/(2**KBIT)                                    0058.000
                      ERATE(1)=(DEDOT*DCOS(DHDR(11)))/DRATE                     0059.000
                      ERATE(2)=0                                                0060.000
                      ERATE(3)=-(DEDOT*DSIN(DHDR(11)))/DRATE                    0061.000
                C                                                               0062.000
                C     WRITE OUTPUT HEADER                                       0063.000
                C                                                               0064.000
                      PNVC=0                                                    0065.000
                      DOHDR(12)=DPLTME                                          0066.000
                      DOHDR(13)=BIT                                             0067.000
                      DOHDR(14)=DTRUN                                           0068.000
                      DOHDR(15)=DTRUN                                           0069.000
                      DO 10 J=1,11                                              0070.000
                   10 DOHDR(J)=DHDR(J)                                          0071.000
                      DO 102 K=1,3                                              0072.000
                  102 DOHDR(15+K)=DHDR(14+K)                                    0073.000
                      DOHDR(11)=KBIT                                            0073.100
                      DOHDR(19)=DHDR(18)                                        0074.000
                      DTTIME=0                                                  0074.010
                      DINTD=0                                                   0074.020
                      KB=DHDR(19)                                               0074.030
                      DOHDR(20)=DHDR(19)                                        0074.040
                      IF(KB.EQ.0)GO TO 103                                      0074.050
                  111 READ(22,END=110)DHDRC                                     0074.060
                      READ(22)DCVNS                                             0074.070
                  112 READ(22,END=110)SDOUD                                     0074.080
                      GO TO 112                                                 0074.090
                  110 READ(22)CVNS                                              0074.100
                      READ(22)DINTD                                             0074.200
                      READ(22)DTTIME                                            0074.300
                      WRITE(KIT,117)DTTIME                                      0074.400
                  117 FORMAT(' PRESENT RUNNING TIME =',F9.1,1X,'SECONDS'/       0074.500
                     1 ' ENTER 1 TO START TEST PROGRAM, 0 TO SEARCH MORE')      0074.600
                      READ(KIT,2) KCOM                                          0074.700
                      READ(22,END=115)SDOUD                                     0074.800
                  115 IF(KCOM.EQ.0)GO TO 111                                    0074.900
                  103 WRITE(22)SDUMDR                                           0075.000
                      DLAT=DHDR(11)*180.0D+0/DPI                                0076.000
                C                                                               0077.000
                C     INITIALIZE C MATRIX                                       0078.000
                C                                                               0079.000
                      READ(21)DCVNS                                             0080.000
                      WRITE(22)DCVNS                                            0081.000
                      IF(KB.EQ.1) GO TO  116                                    0081.100
                      DO 20 J=1,3                                               0082.000
                      DO 20 K=1,3                                               0083.000
                      IF(J.EQ.K)GO TO 201                                       0084.000
                      CVN(J,K)=0                                                0085.000
                      GO TO 20                                                  0086.000
                  201 CVN(J,K)=2**(KBIT-1)                                      0087.000
                   20 CONTINUE                                                  0088.000
```

```
04124140    FORTRAN       SYSTEMS REAL-TIME MONITOR-7.0

S E L  E X T E N D E D  F O R T R A N  I V  (R E V - 0 / 7 8 S E P 0 8)

TNKTST

                  116   DOUD=0.0D+0
                        DPIV=0.0D+0
                        NMY=(1.0D-2+1.0D-5)/DHDR(1)
                        TMAX=DPLTME/(1.0D-2)
                        PTMAX=DPTIME/DPLTME
                        DTHRES=1.0D+0/(2.0**(BIT))
                         PSCALE=2.0**(KBIT-BIT)
                        DTHRESD=DTHRES*180.D+0/DPI
                        TTIME=0
                        PTIME=0
                        TIME1=0
                        TIME=1.0D-2/DTIME
                        CINT=0
                        LPCNT=25
                        M=1
                C
                C   TOP OF MAIN PROCESSING LOOP
                C
                   24   READ(21,END=101)SIND
                        IF(DTTIME.GE.DTRUN)GO TO 101
                        N=1
                        NMX=0
                   25   NMX=NMY+NMX
                        IF(NMX.GT.251)GO TO 24
                  593   DTTIME=DTTIME+DHDR(1)
                        TIME1=TIME1+1
                        DO 500 J=1,3
                  500   DINTD(J)=DINTD(J)+DIND(J,N)
                        IF(N.GE.NMX) GO TO 594
                        N=N+1
                        GO TO 593
                  594   DO 591 J=1,3
                        CINT(J)=DINTD(J)/DTHRES
                        DINTD(J)=DINTD(J)-CINT(J)*DTHRES
                  591   CONTINUE
                  504   DO 505 J=1,3
                        PIVI(J)=CINT(J)*PSCALE
                  505   CINT(J)=0
                C
                C       CALCULATE EARTH RATE INCREMENTS AND SUBTRACT FROM INPUT
                C
                        TIME1=TIME1*DHDR(1)/DTIME
                        DO 22 J=1,3
                        ERIN(J)=0
                        DO 221 K=1,3
                        CALL MUL32(CVN(K,J),ERATE(K),TEMP,SBIT)
                  221   ERIN(J)=ERIN(J)+TEMP*2
                        CALL MUL32(ERIN(J),TIME1,PINI(J),SBIT)
                   22   PNVI(J)=PIVI(J)-PINI(J)
                        TIME1=0
                C
                C       SET DELTA PNV MATRIX AND UPDATE C MATRIX
                C
                        PNVC=0
```

S E L E X T E N D E D F O R T R A N I V (R E V - 0 / 7 8 S E P 0 8)

TNKIST

```
                PNVC(1,2)=-PNVI(3)
                PNVC(1,3)=PNVI(2)
                PNVC(2,1)=PNVI(3)
                PNVC(2,3)=-PNVI(1)
                PNVC(3,1)=-PNVI(2)
                PNVC(3,2)=PNVI(1)
          C
          C     UPDATE C MATRIX
          C
                IF(KU.EQ.0)GO TO 511
                DO 506 J=1,3
                DO 506 K=1,3
                PNVMS(J,K)=0
                DO 507 L=1,3
                CALL MUL32(PNVC(J,L),PNVC(L,K),TEMP,SBIT)
           507  PNVMS(J,K)=PNVMS(J,K)+TEMP/2
           506  CONTINUE
                DO 508 J=1,3
                DO 508 K=1,3
           508  PNVC(J,K)=PNVC(J,K)+PNVMS(J,K)
           511  DO 509 J=1,3
                DO 509 K=1,3
                CVND(J,K)=0
                DO 510 L=1,3
                CALL MUL32(CVN(J,L),PNVC(L,K),TEMP,SBIT)
           510  CVND(J,K)=CVND(J,K)+TEMP
           509  CONTINUE
                DO 26 J=1,3
                DO 26 K=1,3
            26  CVN(J,K)=CVN(J,K)+CVND(J,K)
          C
          C  CHECK FOR OUTPUT DATA
          C
           530  TTIME=TTIME+1
                IF (TTIME.LE.TMAX)GO TO 501
                TTIME=0
                DCVN1=CVN(1,1)*DCSC
                DCVN2=CVN(2,1)*DCSC
                DOUD(1,M)=DATAN2(DCVN2,DCVN1)*180.0+0/DPI
                DOUD(2,M)=DOUD(1,M)-DIND(4,N)
                SCVN=CVN(3,3)*DCSC
                IF(SCVN.GT.1.0)SCVN=1.0
                DOUD(3,M)=ACOS(SCVN)*180.0/DPI
                DOUD(4,M)=DOUD(3,M)-DIND(5,N)
           534  IF(DOUD(2,M).GE.150.0)GO TO 531
                IF(DOUD(2,M).LE.(-150.0))GO TO 532
                GO TO 533
           532  DOUD(2,M)=DOUD(2,M)+180.0
                GO TO 534
           531  DOUD(2,M)=DOUD(2,M)-180.0
                GO TO 534
          C
          C  CHECK FOR PRINT OUTPUT
          C
```

```
04:24:40    FORTRAN       SYSTEMS REAL-TIME MONITOR-7.0

S E L  E X T E N D E D  F O R T R A N  I V  (R E V - 0 / 7 8 S E P 0 8)

INKTST

      533   PTIME=PTIME+1
            IF(PTIME.LE.PTMAX)GO TO 524
            PTIME=0
            IF(LPCNT.LT.25) GO TO 586
            LPCNT=0
            WRITE(7,580)
      580   FORMAT('1')
            WRITE(7,581)
      581   FORMAT(4X,'BITS',3X,'SAMPLE',10X,'AMPLITUDE',
          1 18X,'PERIOD',13X,'PHASE(1=SIN,0=COS)',3X,'MOTION(1=V,0=N)',
          2 6X,'THRESHOLD'/)
            WRITE(7,582)     MBIT,DHDR(1),(DHDR(J),J=2,7),(DHDR(K),K=15,17),
          1 DHDR(19),DTHRESD
      582      FORMAT(4X,I4  ,7F9.3,2X,3F5.0,11X,F5.0,11X,F10.8,//)
            WRITE(7,583)
      583   FORMAT(5X,'TIME',7X,'C11',7X,'C12',7X,'C13',7X,'C21',
          1 7X,'C22',7X,'C23',7X,'C31',7X,'C32',7X,'C33',3X,
          2 'HEADING',3X,'ERROR',5X,'TILT'/)
      586   DO 587 J=1,3
            DO 587 K=1,3
      587   DCVN(J,K)=CVN(J,K)*DCSC
            WRITE(7,584)DTTIME,((DCVN(J,K),K=1,3),J=1,3),(DOUD(L,M),L=1,3)
      584   FORMAT(1X,F9.3,9F10.5,F11.4,2F9.4)
            DO 520 J=1,3
            DT=0
            DO 521 K=1,3
      521   DT=DT+DCVN(J,K)**2
      520   DCVNR(J)=DSQRT(DT)
            DO 522 K=1,3
            DT=0
            DO 523 J=1,3
      523   DT=DT+DCVN(J,K)**2
      522   DCVNC(K)=DSQRT(DT)
            DCVND=DCVN(1,1)*DCVN(2,2)*DCVN(3,3)+
          1       DCVN(1,3)*DCVN(2,1)*DCVN(3,2)+
          2       DCVN(1,2)*DCVN(2,3)*DCVN(3,1)-
          3       DCVN(1,3)*DCVN(2,2)*DCVN(3,1)-
          4       DCVN(1,2)*DCVN(2,1)*DCVN(3,3)-
          5       DCVN(1,1)*DCVN(2,3)*DCVN(3,2)
            WRITE(7,585)DCVND,(DCVNP(J),J=1,6)
      585   FORMAT(5X,7F13.8)
      512   LPCNT=LPCNT+1
      C
      C  CHECK FOR OUTPUT RECORD
      C
      524   M=M+1
            IF(M.LE.500) GO TO 501
            M=1
            WRITE(22)SDOUD
            IF(DTTIME.GE.DTRUN)GO TO 104
            DOUD=0
      501   N=N+1
            GO TO 25
       101  WRITE(22)SDOUD
```

```
04:24:40    FORTRAN       SYSTEMS REAL-TIME MONITOR-7.0

S E L  E X T E N D E D  F O R T R A N  I V  (R E V - 0 / 7 8 S E P 0 8)

TNKIST

                    104   ENDFILE 22                                                  0258.000
                          WRITE(22)CVNS                                               0259.000
                          WRITE(22)DINTD                                              0259.100
                          WRITE(22)DTTIME                                             0259.200
                          ENDFILE 22                                                  0260.000
                          WRITE(KTY,150)                                              0261.100
                    150   FORMAT(' ENTER 1 TO REWIND INPUT TAPE, 0 TO LEAVE AS IS'/)  0261.200
                          READ(KTY,2)KRW                                              0261.300
                          IF(KRW.EQ.1)GO TO 152                                       0261.400
                          READ(21)DCVNS                                               0261.500
                          READ(21,END=151)SIND                                        0261.600
                    152   REWIND 21                                                   0262.000
                          ENDFILE 22                                                  0262.010
                          REWIND 22                                                   0262.020
                          READ(22)SDOHDR                                              0262.030
                          DOHDR(15)=DTRUN                                             0262.040
                          REWIND 22                                                   0262.050
                          WRITE(22)SDOHDR                                             0262.060
                    151   CONTINUE                                                    0262.100
                          CALL EXIT                                                   0263.000
                          END                                                         0264.000
```

04124140 FORTRAN SYSTEMS REAL-TIME MONITOR-7.0

SEL EXTENDED FORTRAN IV (REV-0/78SEP06)

MUL32

```
1                          SUBROUTINE MUL32(I,J,K,L)                    0265.000
2                          I1=I                                         0266.000
3                          I2=J                                         0267.000
4                          I3=L                                         0267.100
5                          INLINE                                       0268.000
6                          LW 7,I1                                      0269.000
7                          LR 5,I2                                      0270.000
8                          LNW 3,I3                                     0270.100
9                          MPR 5,6                                      0271.000
10                   )400  SLLD 6,1                                     0272.000
11                         BIB 5,)400                                   0272.100
12                         STW 6,I3                                     0273.000
13                         ENDI                                         0274.000
14                         K=I3                                         0275.000
15                         RETURN                                       0276.000
16                         END                                          0277.000
```

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